MATHEMATICAL OPTIMIZATION

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MATHEMATICS COLLOQUIUM SERIES
BACKGROUND INFORMATION

• **Mathematical Optimization** - the selection of the best component from a set of available options

• **Optimization Problems** consist of finding the maximum or minimum of a real function, known as an optimal solution

• **Often involves a constraint**

• **Most widely used in the areas of Mathematics, Computer Science, and Operations Research**
MATHEMATICAL APPLICATIONS

• Calculus of variations seeks to optimize an action integral over some space to an extremum by varying a function of the coordinates.

• Global optimization - the development of deterministic algorithms that are capable of guaranteeing convergence in finite time to the actual optimal solution of a nonconvex problem.

• Mathematical approaches to solving optimization problems include classical, linear & nonlinear programming, and game theory.
BUSINESS APPLICATIONS

- Optimal allocation of resources lies at the heart of the science of economics.
- Consumers are assumed to maximize their utility, while firms are usually assumed to maximize their profit.
- Asset prices, trade theory, and the optimization of market portfolios are also modeled using optimization theory.
- Macroeconomists build dynamic stochastic general equilibrium (DSGE) models that describe the dynamics of the whole economy.
The relationship between the optimal indifference curve $U(x, y) = C$, where $C = U(18, 8)$ and the budgetary constraint is $20x + 30y = 600$

The relationship between the budgetary constraint and the level curve for optimal sales
REAL WORLD EXAMPLE: DISNEY WORLD
• Crowd Calendar shows how busy each Disney theme park is
• Attendance across different weeks, months and seasons
• Customers can plan a park visit and avoid crowds with the help of:
  • Each park’s opening and closing times
  • The park’s Extra Magic Hours schedule
  • Any special events that might affect your visit
• Shows yesterday’s results - their predictions versus what actually happened
SUMMER 2016 AT DISNEY

- Graph data based on standby waits, posted waits, and people in line between 10 a.m. and 5 p.m. (the peak time for crowds).
- Most major attractions at the Animal Kingdom has had a wait time drop in 2016.
- Attendance is lower for Epcot, Disney’s Hollywood Studios, and the Animal Kingdom, but higher at the Magic Kingdom, versus the same period in 2015.
- Overall, attendance is slightly lower throughout Walt Disney World.
MAIN ALGORITHM

- **Traveling Salesman Problem (TSP)**
  - Optimal route
  - Better solution = cheaper solution
- **Time Dependent Traveling Salesman Problem (TDTSP)**

  The cost to travel from one city to another depends on:
  - The distance between cities
  - Time of day of the travel
ALGORITHM: AVOIDING LONG LINES

- \( Q = \) COMPUTER TIME TO COME UP WITH A RESULT
- \( R = \) THE SET OF ALL RIDES YOU WANT TO RIDE
  - \( r = \) SPECIFIC RIDE IN \( R \)
- \( E_{ij}^t = \) WALK FROM RIDE \( i \) TO \( j \) AT TIME \( t \)
- \( \text{Start at the entrance and run a time-dependent Nearest Neighbor algorithm for each ride in } R \)
  - \( \text{each } r_i \text{ in } R \text{ is the ride visited after entering the park} \)
- \( \text{Save the set of all edges found in the paths into } S \)
- \( \text{Create a small number – random TSP paths for your rides – just put your rides in } R \text{ in any random order to start with. For each path in } P \text{, calculate the “cost” of the path} \)
ALGORITHM CONTINUED

• **While (we still have time according to Q)**

• **Pick 2 paths (parents) from P using tournament selection**

• **Pick a genetic operator such as:**
  
  • **Random Mutation**
  
  • **Time-Dependent Random Mutation**
  
  • **Lin-Kernighan**
  
  • **2-opt**
  
  • **Cycle Crossover**
  
  • **Brute Force Permutation**
  
  • **Fast Pass Mutation**

• **Apply the chosen operator to the parents. The path that is created by this operator and the parents is called the child**

• **Calculate the cost of the child**

• **If the child’s cost is less than the cost of the worst path in P:**
  
  • **Delete the worst path in P**
  
  • **Add child to P**

• **If we’ve gone a really long time without adding a child to P:**
  
  • **Delete all but the 1 best path in P**
  
  • **Create new, random paths for all of the remaining space in P**

• **Done // While (we still have time..)**

• **Send the results back to the server**
HOW MANY POSSIBILITIES ARE THERE?

- **Magic Kingdom** has 43 attractions, ranking between the game of **Chess** and **Go** in terms of complexity.

- Takes into considerations food places and shows.

- Variations for different times of the day or year.

- Takes a lot of computing power.

- There are $10^{170}$ possible moves in the game *Go*, while only $10^{80}$ number of atoms in the observable universe.

<table>
<thead>
<tr>
<th>Game</th>
<th>Ways to Play</th>
<th>Like a Touring Plan with</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tic-Tac-Toe</td>
<td>$31,896$</td>
<td>$8$ attractions</td>
</tr>
<tr>
<td>Connect 4</td>
<td>$4.6 \times 10^{12}$</td>
<td>$15-18$ attractions</td>
</tr>
<tr>
<td>Checkers</td>
<td>$5 \times 10^{20}$</td>
<td>$21-22$ attractions</td>
</tr>
<tr>
<td>Chess</td>
<td>$10^{40}$ to $10^{50}$</td>
<td>$35-42$ attractions</td>
</tr>
<tr>
<td>Go</td>
<td>$10^{170}$</td>
<td>$108$ attractions</td>
</tr>
</tbody>
</table>
CALCULUS OF VARIATIONS

• The feasible points that satisfy the constraint form a polygon
• The edges of a polygon – edges of the park area
• Each ride is similar to a vertex on a polygon
• The extrema occur at the vertices
• Many similar problems involve linear programming
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