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Vagueness and Fuzzy Logic

by Darren Hibbs

Some terms are vague. A vague term is one that foils our attempts to establish whether it applies in some cases. Consider the term “bald.” There are cases where it is clear that a person is bald and cases where it is clear that a person is not bald. But there are some cases where it is not clear whether the individual in question qualifies as “bald.” Many of the terms we use in ordinary situations have this feature. Terms such as “city,” “tall,” “middle aged,” and “rich,” admit borderline cases where it is not clear that the term applies or fails to apply. Moreover, uncertainty about the applicability of a vague term does not seem to be a result of ignorance on the part of the language user. For example, one may know the exact number of hairs on a head or the exact number of people who live in a community, yet still be unsure about whether the head is bald or whether the population constitutes a city.

Vagueness poses a number of philosophical problems. For example, vagueness appears to threaten a basic principle of classical logic known as “bivalence.” Bivalence is a theory about propositions. Propositions are meaningful assertions that something is the case. According to the principle of bivalence, every proposition is either true or false, but not both. The intuition supporting bivalence is straightforward—an assertion that some state of affairs obtains is either accurate or inaccurate. But consider the proposition “That person is bald.” If the principle of bivalence is true, the proposition must be either “true” or “false.” But if the person in question is a borderline case of baldness, the proposition “That person is bald” resists classification as either “true” or “false.” Examples of this sort lead some critics of classical logic to reject bivalence in favor of alternative methods of characterizing the truth-values of propositions.1

The problem of vagueness is also instrumental in the construction of logical paradoxes. A logical paradox is an apparently sound argument that contains an apparently absurd or false conclusion. Paradoxes generate problems in logic because sound arguments are not supposed to entail absurd or false conclusions. A classic exposition of a vagueness-related paradox is attributed to Eubulides, a 4th-century philosopher from Miletus, who formulated the following
paradox based upon the vagueness of the term “heap.” The version presented here is just one of many ways to characterize the paradox. Suppose there is a heap of 100,000 kernels of corn. If one kernel is removed from the heap, does the heap of corn still exist? The intuitive answer is “yes.” If another kernel is removed, does the heap of corn still exist? Again, the intuitive answer is “yes.” If 100,000 kernels is a heap of corn, then 99,998 kernels is also a heap of corn. Suppose that this process continues with one kernel at a time being removed until all the kernels are gone and the heap has vanished. The question is: at what point does the heap no longer exist? There are only two possible answers:

(1) There is no point that marks the demise of the heap.

(2) There is a point that marks the demise of the heap.

Option (1) entails that the heap never disappears. But this is false, since at the final stage of the process, there are no kernels left to compose a heap. According to option (2), the existence of a heap must depend upon the presence or absence of one kernel of corn. But this is absurd. A single kernel of corn is not a heap-making entity. The situation presents a paradox because a simple chain of reasoning (if “x” is removed, “y” is still a heap, etc.) leads to two possible conclusions that are robustly implausible. Vagueness is present in the case because a heap is another term that admits borderline cases. It is simply unclear whether a heap is present in some circumstances, even if one knows the number of components involved (kernels, in this case).

This sort of paradox can be mapped onto other cases where the numerical identity (or sameness) of unique objects over time is susceptible to vagueness. Numerical identity is distinguished from qualitative identity. Objects “x” and “y” bear a relation of qualitative identity when both objects possess the same qualities. For example, imagine two billiard balls that are both composed of the same type of material and possess the same dimensions, weight, and color. The billiard balls are qualitatively the same since they possess the same qualitative features. But they are not numerically identical because there are two billiard balls rather than one. That is, one ball could be destroyed while the other continues to exist. Numerical identity would be the target if one were to ask whether one of the billiard balls was the “same” ball that was used in a game a week earlier. This question is not about whether one of the balls possesses the same qualitative features as the one used earlier. This question is about whether one of the two balls is the one that was used a week earlier. We attribute numerical identity to many things on a regular basis. The people one knows, the car one drives, one's dwelling, a familiar tree seen on the way to work, etc. are all considered numerically the same over time, despite the qualitative changes that these kinds of entities undergo over time. But the numerical identity of objects over time may be obscured in cases where vagueness is present.

Let's consider a scenario involving the gradual disassembly of the original sculpture, Venus de Milo. Suppose that a small particle the size of a grain of sand was removed from the sculpture. Let us call this altered version of the sculpture Venus*. Is Venus* numerically the same as the original Venus? As in the case of the heap, the removal of one particle of that size would not, intuitively, amount to the destruction of the original statue. Although there is a trivial qualitative change, it is surely the case that Venus* is numerically the same as the original Venus. We may continue the process of disassembly by removing another minute particle
resulting in Venus**. If Venus* is numerically identical to the original Venus, then surely Venus** is identical to Venus*, since Venus** is the result of the same identity-preserving procedure that brought about Venus*. Following the example of the heap, the process of disassembly may proceed in a piecemeal fashion until there are no particles of the original Venus remaining. At each stage of the disassembly, the relevant question will be “Is the original Venus still existing?” Thus, we are facing a familiar choice:

(1) The original Venus exists throughout the process of disassembly.

(2) There is a point in the process where the removal of one particle entails that the original Venus no longer exists.

Option (1) is flatly wrong, since it commits one to the view that the original Venus is identical to nothing. Option (2) commits one to the view that the continued existence of the statue is decided on the basis of the presence, or absence, of one minute particle. Thus, option (2) is intuitively absurd. Vagueness plays a role in this case in the sense that there are stages in the process of disassembly where it is not clear whether the statue is the same as the original.

The puzzles generated by vagueness raise fundamental questions about the nature of propositions and the soundness of classical logic. As noted above, some critics of classical logic respond to the problem of vagueness by rejecting the principle of bivalence in favor of alternative accounts of assigning truth-values to propositions. Again, according to the principle of bivalence every meaningful proposition must be either true or false, but not both. Non-propositional linguistic expressions do not possess a truth-value. For example, questions, exclamations, and gibberish are neither true nor false since they do not assert that something is or isn't the case. But a proposition possesses one, and only one, truth-value regardless of whether we know (or are capable of knowing) its truth-value. For example, the proposition “God exists” is either true or false, regardless of whether anyone knows (or could know) that the proposition is true or false. If bivalence is true, the proposition “That person is bald” must also be true or false since it is a declaration that something is the case. But this is problematic. In some cases, one can know all the relevant information (the number of hairs, their arrangement, etc.) yet still be unsure about whether the assertion is true or false. Thus, the problem does not appear to be the result of epistemic limitations.³ It may be the case that propositions employing vague terms simply defy the bivalent categorization of classical logic.

One response to this problem is to reject bivalence and propose an alternative account of truth-value assignments to propositions. For example, one may argue that some propositions do not have a truth-value or that there may be more than two truth-values for propositions. The latter strategy utilizes a many-valued or polyvalent method of assigning truth values to propositions. One form of polyvalence is employed by proponents of “fuzzy logic.”⁴ Fuzzy logic posits a range of possible truth-values for propositions. The range of possible truth values can be represented numerically. For example, a proposition that is completely false is assigned a value of (0) and a proposition that is completely true is assigned a value of (1). But some propositions are neither completely true nor completely false, so their truth-value assignment would be a number between (0) and (1). The numerical truth-value can be fine-tuned to any decimal place. Consider the proposition “That person is bald.” If the person's head is completely hairless, the
proposition is assigned a value of (1) since the statement is completely true. If the person's head is densely covered with hair, the proposition is assigned a value of (0) since the claim is completely false. But if the head in question is a borderline case, the truth-value of the proposition might be something like (.432). The assignment of (.432) indicates that the proposition is sort of true but mostly false. But in other cases the proposition might receive a value of (.745), which indicates that the proposition is mostly true and slightly false.

Proponents of fuzzy logic argue that a polyvalent method of assigning truth-values offers a more precise classification of propositions infected by vagueness when compared to the bivalent method. The application of the fuzzy method to the paradox of the heap and the case of the statue is straightforward. Consider the following propositions:

(a) The heap of corn is still present.
(b) The Venus de Milo is still present.

According to the fuzzy logic approach, it is not necessary to view each of these propositions as perfectly true or false. Each proposition might be more or less true, given the circumstances. For example, about half way through the process of disassembling the statue, the truth-value of (b) would reflect the degree of departure from perfect truth to some value that is (>0) but (<1). Thus, instead of being unable to assign a truth value at all (which is the problem for the advocate of bivalence), the fuzzy logician is in a position to capture the gradual erosion of truth at each stage in the process of disassembly.

The fuzzy logic approach to the problem cases above is intuitively attractive, but closer scrutiny of the fuzzy method reveals serious difficulties that merit our attention. I will briefly explain two of these difficulties. The first problem is about the failure of a polyvalent approach to addressing the issue of vagueness. The second problem is related to the concept of truth.

The motivation for adopting a many-valued approach in cases of vagueness is to avoid the embarrassing quandary associated with bivalence in borderline cases. But the many-valued approach fails to avoid the problem of vagueness in borderline cases in the same manner as the bivalent approach. For example, if the fuzzy logician assigns a truth-value of (.432) to the proposition “That person is bald”, a reasonable question would be to ask how the (.432) state of baldness is distinguished from the (.433) or the (.431) state of baldness. Given an appropriately middling state of baldness, there doesn't seem to be a non-arbitrary way of choosing one of these truth-values over the other. That is, there doesn't seem to be a fact about the state of baldness that signals the correctness of one value but not the others. But this is the same problem that confronts the proponent of bivalence in borderline cases. The vagueness that attends borderline cases of “bald” and “not bald” is simply relocated to the border of the finer distinction between (.432) and (.433) degrees of baldness. Since borderline cases apply within the fuzzy system also, the polyvalent method of assigning truth-values is susceptible to the same puzzles and paradoxes that confront the bivalent method.

The proponent of fuzzy logic may respond to this objection by pointing out that it is perhaps less worrying to be unsure about the difference between (.432) and (.433) baldness than it is to
be unsure about the distinction between “bald” and “not-bald.” That is, fuzzy logic provides us with a more precise measurement of baldness even though vagueness has not been eliminated from the picture. But this result can be achieved without dispensing with bivalence. We may simply stipulate very precise conditions for baldness, heaps, etc. so that we will be able to assign truth-values for propositions in some of the formerly troublesome cases. We do not bother with this in ordinary linguistic practice. Why? We do not need very precise definitions for baldness or heaps in everyday discourse. Our ability to communicate effectively simply does not require rigorous conceptual precision in most circumstances. Furthermore, vagueness is desirable for some communicative purposes. If I suggest that some proposed activity be postponed until “later,” a person on the lookout for vagueness might note that it is difficult to pinpoint what I mean by “later.” But this might be due to the fact that I do not know exactly what I mean by “later.” I only know that the activity is not desirable now, but presumably will be desirable at some unknown point in the future. Thus, vagueness is sometimes an essential component of communication.

A second problem involves the concept of truth itself. The concept of truth as it is employed in formal logical systems is distinct from the concept of truth in the metaphysical sense. The former concept of truth is merely a function of the rules or axioms of a logical system. The rules of a logical system do not tell us whether a given proposition is actually true or false, or what it means to assert that a proposition is true or false in general. For example, classical logic defines a conjunction (“P and Q”) as “true” if and only if “P” and “Q” are true individually. This rule is not about the actual truth-value of “P” or “Q” nor does the rule supply a theory about what it means for “P” or “Q” to be true individually. Thus, formal logical systems are only interested in truth to the extent that pre-assigned truth-values are preserved or altered in certain procedures. But an understanding of the formal role of truth within a logical system does not address the metaphysical question of truth. What is meant when we assert that a proposition is true? There are several competing accounts of truth that are designed to answer this question. These theories are, in part, guided by our use of the term “true” in natural languages. For example, to assert that some proposition “P” is true may mean that the content of “P” accurately represents some relevant portion of reality, or that it is useful to believe that “P” is the case, or that “P” coheres with a set of mutually supporting beliefs. The nature of these disagreements is not the issue in the present context. The current issue is about the desired relationship between the notion of truth in the logical and metaphysical senses.

It is natural to assume that the concept of truth employed in a formal logical system would dovetail (in some relevant way) with a metaphysical account of truth that is grounded in ordinary usage. That is, our formal notion of truth ought to represent at least some aspects of our informal notion of truth. But when we consider the fuzzy notion of truth, it is not clear what is meant when one asserts that a proposition possesses a truth-value of (.432)—i.e. that it is “partially true.” Although we often employ the notion of “partial truth” to evaluate theories, testimony, etc., when pressed about what we mean by “partially true” those general assessments can be analyzed into component propositions that are individually true and false in accordance with bivalence. For example, we may analyze the testimony of a witness in a criminal trial as being only partially true in the sense that the witness issued some statements that are true and some statements that are false. Individual statements themselves may be subject to a similar analysis. But the notion of partial truth is basic to fuzzy logic. If all propositions with a truth-value ranging
from (>0) to (<1) could be analyzed into components that are either (1) or (0), then bivalence has not been eliminated. Thus, some propositions just are (.432) true irreducibly. But it is difficult to grasp what this could mean without analyzing the proposition into component parts that are either true or false.

However, the fuzzy logician might respond by arguing that this complaint is not to the point. Logical systems are idealized languages that need not conform to the informal customs of ordinary language users. In particular, symmetry with natural languages is not a necessary condition for developing a satisfactory formal account of propositional truth-values. A proponent of fuzzy logic may acknowledge that in ordinary language we analyze partial truths into component propositions that are either true or false, yet argue that this is irrelevant since formal logic is not concerned with ordinary language. Thus, the notion of irreducible partial truths should not count as evidence against the adequacy of the polyvalent approach. But this maneuver generates a dilemma for the fuzzy logician. If formal logic is an idealized language that is not beholden to ordinary language, then the initial motive for renouncing bivalence is no longer operative (i.e. cases involving the application of vague terms). That is, bivalence works perfectly well in classical logic if the only propositions employed are those that are either true or false. Trouble arises only in cases where a bivalent logical apparatus is applied to real world cases where at least one proposition cannot be assigned a truth-value. Alternatively, if the fuzzy logician argues that the polyvalent approach captures ordinary language more precisely than bivalence, we may point to fuzzy logic’s failure to eliminate vagueness and its adherence to the dubious concept of irreducible partial truth that lies outside of ordinary language. Thus, regardless of whether fuzzy logic is regarded as an idealized language or as a formal adjunct to ordinary language, there is no reason to favor polyvalence over bivalence.

Of course, these considerations do not show that fuzzy logic endorses false claims or that it is an incoherent theory. Nor do these considerations show that there are no problems with bivalence. The adoption of a logical system involves trade-offs. As noted above, vagueness is an essential aspect of ordinary language. But the vagueness of ordinary language appears to be incommensurable with the desired precision of a formal language in logic. So what does this mean with respect to the relationship between the formal language of a logical system and ordinary language? For classical logic, it means that the “sweet simplicity” of bivalence is cherished at the expense of facing the bitter reality of vagueness. But the cost of replacing bivalence with fuzzy logic is greater since the problem of vagueness is retained and a suspicious concept of truth is added to the bill. Given these considerations, the case for revoking bivalence in favor of fuzzy logic is not persuasive.

Endnotes

1. The term “classical logic” refers to the logical systems developed by Gottlob Frege, Bertrand Russell, and others during the late 19th and early 20th centuries. Bivalence was also accepted by many ancient and medieval logicians. For a general account of alternative logical systems that reject bivalence, see An Introduction to Non-Classical Logic: From If to Is, by Graham Priest (Cambridge University Press, 2008); and Deviant Logic, Fuzzy Logic, by Susan Haack (University of Chicago Press, 1996).
2. For logical paradoxes in general, see *A Brief History of the Paradox*, by Roy Sorensen (Oxford University Press, 2003). For the “heap” version of the sorites paradox, see *Vagueness: A Reader*, edited by Rosanna Keefe and Peter Smith (MIT Press, 1999), especially chapters 2, 7, 10, 12, and 15.

3. Some philosophers argue that vagueness is a result of epistemic failure. See *Vagueness and Contradiction*, by Roy Sorensen (Oxford University Press, 2001); and *Vagueness*, by Timothy Williamson (Routledge, 1994).

4. See Priest and Haack, note 1, for general accounts of fuzzy logic.

5. See *Vagueness and Degrees of Truth*, by Nicholas J. J. Smith (Oxford University Press, 2009), chapter 5.


7. There are still other options. For detailed accounts of standard theories of truth, see *Truth*, edited by Simon Blackburn and Keith Simmons (Oxford University Press, 1999).