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Abstract

A challenge in mathematics education research has been to document the complex nature of collective mathematical learning. This paper describes a method of data analysis that offers a visual representation of collective discourse during mathematical tasks. Using data extracts from a study of small groups in a middle years classroom, I color code collective utterances to create a "tapestry," a type of transcript that offers researchers the ability to move between individual and collective planes of focus during analysis. The nature of collective thinking is revealed by tapestries, including how utterances bump against each other, the role of utterances evolves as the context of discussion changes, and the potential for self-structuring within collective discourse.

Keywords

Collective Discourse, Mathematics Education, Small Groups, Middle Years Students, Collective Understanding

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A challenge in mathematics education research has been to document the complex nature of collective mathematical learning. This paper describes a method of data analysis that offers a visual representation of collective discourse during mathematical tasks. Using data extracts from a study of small groups in a middle years classroom, I color code collective utterances to create a "tapestry," a type of transcript that offers researchers the ability to move between individual and collective planes of focus during analysis. The nature of collective thinking is revealed by tapestries, including how utterances bump against each other, the role of utterances evolves as the context of discussion changes, and the potential for self-structuring within collective discourse.
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While there have been a growing number of studies exploring the collective nature of mathematical learning (e.g. Bowers & Nickerson, 2001; Clark, James, & Montelle, 2014; Cobb 1999; Davis & Simmt, 2003; Francisco, 2013; Martin, Towers, & Pirie, 2006; Rasmussen & Stephan, 2008; Rasmussen, Wawro, & Zandieh, 2015; Yackel & Cobb, 1996) there is still a need for new analytical models to document the emergent nature of collective understanding (Davis & Simmt, 2008; Francisco, 2013; Towers & Martin, 2015). In this paper, I propose a method of data analysis that offers a visual representation of collaborative discourse through the creation of a "tapestry" style transcript. Using a dialogistic framework, I will discuss how tapestry transcripts developed from data from a study of small groups in middle years classrooms engaged in mathematical tasks (Armstrong, 2013) offer researchers a new way of seeing collective discourse by providing the ability to move between two planes of focus during analysis. I also discuss the nature of collective thinking revealed by tapestries, including how utterances "bump" against each other, how the role of utterances evolves as the context of discussion changes, and the potential for self-structuring within collective discourse.

Studying Collective Thinking

Many researchers[1] have studied group learning over the years (e.g. Cohen, 1994; Webb et al., 2009) in the interest of increasing the effectiveness of small groups in mathematics classroom settings (e.g. Mercer & Littleton, 2007; Rezitskaya et al., 2009). Although in casual conversation one might describe what a certain classroom group thinks, it has been challenging for researchers to conceptualize the group as a unit of analysis, even when the group is small. For instance, if one follows an acquisitionist view (Sfard, 1991) where the mind is seen to function as a container and learning is a matter of pieces of knowledge being transmitted from the teacher's mind, acquired by the student, and then stored in his or her mind, then the idea of group learning makes no sense. Once the group breaks up, as it inevitably must, and the members go their different ways, where does the group's learning go? There is no permanent structure – for instance, a group brain – to contain it. Even when considering learning as

[1] Johnson and Johnson (2009) note that more than 1,200 studies about social interdependence have taken place in the past eleven decades.

adapting to new circumstances, rather than storing chunks of knowledge, the concept of group learning is still “a difficult, counter-intuitive way of thinking for many people” (Stahl, 2006, p. 16) due to the strong association of cognition with an individual psychological process.

Some researchers have tackled this challenge of studying collective learning by considering the development of classroom socio-mathematical norms (Yackel & Cobb, 1996). Bridging the apparent gap between individual and group, the concept of *taken-as-shared* involves the meaning that develops between individuals through their social interactions, and evolves as students make adaptations “which [eliminate] perceived discrepancies between their own and others’ mathematical activity while pursuing their goals” (Cobb, Yackel, & Wood, 1992, p. 118). Voigt writes that the concept of taken-as-shared goes beyond suggesting that individuals can come to agree that they have ascribed the same meaning to an idea: “From the observer’s point of view, the meaning of taken-as-shared is not a partial match of the individual’s constructions, nor is it a cognitive element. Instead, it exists in the process of interaction”(1996, p. 34). It is present neither in the group, nor the individual, but in the moments in which the individuals are negotiating and that the group itself is acting as one.

Theories like this seek to explain how sociomathematical norms develop over a long period of time, suggesting that these norms become constant and stable once they have been established (Yackel & Cobb, 1996). Other researchers have been more interested in considering the nature of collective behavior. Kilgore, who discusses learning in social movements, notes the presence of emergent behavior in collective learners (1999), and Davis and Simmt (2003) propose that mathematics classrooms, and the smaller groups within them, are “adaptive and self-organizing” complex learning systems (p. 138).

In recent years, some researchers (Francisco, 2013; Martin & Towers, 2009; Martin, Towers, & Pirie, 2006; Towers & Martin, 2009, 2015; Towers, Martin, & Heater, 2013) have used an improvisational framework to explore how the interactions of group members unfold as they collaboratively engage with mathematical tasks. Towers and Martin have worked with Pirie-Kieren theory – which views mathematical understanding as a dynamic process of recursive growth and change (Pirie & Kieren, 1994). They characterize collective mathematical understanding as developing through *improvisational coaction*, “a process through which mathematical ideas and actions, initially stemming from an individual learner, become taken up, built upon, developed, reworked and elaborated by others, and thus emerge as shared understandings for and across the group, rather than remaining located within any one individual,” noting that it is a specific type of interaction that “requires mutual, joint action” (Martin & Towers, 2009, p. 4).

Francisco (2013) writes that when studying collaborative groups, “[c]apturing such complexity is an ongoing challenge for researchers, requiring creative theoretical frameworks that can best account for the intended level of complexity” (p. 436). Davis, Smith, and Leflore (2008) have graphed group conversations to show the presence of strange attractors. As mentioned, Towers and Martin (2006, 2015) have employed Pirie-Kieran diagrams to illustrate the folding back of collective understanding. Towers and Martin (2015) have also sought to build on their work by using a transcriptional device where the conversational turns of a small group’s discussion are run together, rather than being on separate lines, and the words are color-coded according to who is talking. This, they argue, has the advantage of showing how “a single coherent sentence that moves the mathematics forward can be formed by multiple voices” (Towers & Martin, 2015, p. 254), evidence of improvisational coaction. As well, they suggest that the colour coding could offer a visual tool for global analysis of who is speaking when and how much. In discussing their transcription device, Towers and Martin (2015) note that, in terms of observing and analysing individual and collective understanding,

> qualitative analysis methods evident in the literature have remained somewhat limited, typically involving direct and painstaking transcription of audio-recorded or videorecorded data and its faithful re-presentation in accurate and elaborate detail. We have seen few experimental and/ or innovative uses of transcription data in the literature. (p. 254)

In this paper, I will outline a method by which researchers can move between the individual and collective levels, and which will provide a tapestry document that will help researchers visualize how collective utterances weave together in the course of a problem solving discussion.

A Dialogical Approach to Collective Discourse

As researchers, we set boundaries all of the time, not only in considering our data, but in collecting it as well. We decide whom to study, where and when to study them, and (sometimes) what they are doing while we study them. We pick artifacts to gather. We choose what technology to use in the collection process – video, audio, chat room – where to position it, how long to let it run (Jordan & Henderson, 1995; Pirie, 1996). And in all these decisions, something is always left out. What we gather, whether they be audio-recordings, video-recordings, photographs, field notes, interviews, all of these are (re)constructions of events that can never be fully captured, never be fully experienced, even by the participants of the events themselves. The transcribing and the coding of this data provide yet another layer of interpretation (Jordan & Henderson, 1995). Even quantitative coding, which somehow seems more scientific, has this bias (Hammer & Berland, 2014). In short, once we have set the boundaries of our study, we are no longer considering "reality" (Osberg, Biesta, & Cilliers, 2008).

Accepting and acknowledging this bias enables researchers to then move ahead in dealing with the complexities of capturing collective discourse. One benefit for researchers who study groups is that group members must make their ideas public to one another in order to be understood (Engeström, 1994) – and thus public to the researcher as well (McDermott, Gospodinoff, & Aron, 1978). Stahl (2006) argues that the group discourse may be considered to represent its thinking:

> [W]hen we say that a group thinks, we are not postulating the group as a unitary physical object but are focusing on the unity of the group's discourse: the fact that effective collaborative discourse is best understood at the level of the group interaction rather than by focusing on the contributions of individual members. The group discourse has a coherence, and the references of the words within it are densely, inextricably interwoven. (p. 399)

Studying collective discourse involves working with the whole of the conversation as it evolves while still appreciating the threads of contributions that make it up. In this dialogic space, it is the "constitutive difference" between these levels that brings them both into existence (Wegerif, 2010). Rogoff (1995) writes of the interactive planes of focus the researcher encounters, which she considers,

> not as separate or as hierarchical, but as simply involving different grains of focus with the whole sociocultural activity. To understand each requires the involvement of the others. Distinguishing them serves the function of clarifying the plane of focus that may be chosen for one or another discussion of processes

> in the whole activity, holding the other planes of focus in the background but not separated. (pp. 141-142)

As researchers, if we choose our plane of focus to be the speech act itself, we can set boundaries to identify individual utterances.[2] In doing so, however, we must realize that it is only through the definitions we set out that the utterance is isolated; it cannot exist on its own. Using the idea of a rope to illustrate this situation, McDermott (1996) writes,

> It is not just that the fibers are analytically unavailable when one is focusing on the rope, it is that half the fibers do not exist except in contrast to other fibers and other parts of the background. All parts of the system define all the other parts of the system. Without the background, there are neither ropes nor fibers. (p. 275)

An utterance is linked to the past in that it is a response to another utterance. This other utterance might be something that has just occurred in the group's ongoing conversation, or has taken place in the day or week or month or year – there are no time limits. Nor are there any limits to what it is that is recalled. It might be something spoken, a written text, a physical experience, a visual image, or it might be within an internal dialogue the subject has been having with herself. This adds to the researcher's challenge. Mercer and Littleton (2007) write,

> A profound problem for researchers wishing to understand how language is used to jointly construct knowledge (and, indeed, with understanding how conversational communication works at all) is inferring what knowledge resources speakers are using. Speakers may make explicit references to shared past experience or other types of common knowledge, but they often invoke such historical, temporal resources only implicitly. Observable features of interactions are likely to have unobservable determinants in the histories of the individuals, groups and institutional systems involved. (p. 121)[3]

An utterance is a response to what has been, or what is currently, happening, and it is also connected to the future, in that it is formed in anticipation of an impending utterance (Bakhtin, 1981).

Considering the dialogicality of a situation also means recognizing that an utterance does not belong to the one who wrote/said/gestured it. Bakhtin (1981) writes, "The word in language is half someone else's. It becomes one's 'own' only when the speaker populates it with his own intentions" (pp. 293-294). Thus, the "conversation" of a group "is crisscrossed by other places and temporalities, by absent third parties, who may express their voice through the participants' discourse" (Grossen, 2009, p. 266) and also by the uptake and reuptake of individual threads of ideas. One might envision the utterance not as a link in a linear chain of threads, but as a part of a fabric that comes from the past and stretches into the future. This fabric is one with ripples spreading outwards from each little change that occurs as the multiple threads of linked discourses affect one another.

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[2] An utterance is "an uninterrupted chain of spoken or written words not necessarily corresponding to a single or complete grammatical unit" (Barber, 1998, p. 1602).

[3] Barnes and Todd (1995) argue that it may even more challenging for those researchers who are observing a group that has a history of working together. "To take an extreme example, some long-standing groups generate catchphrases which for them carry implications which are closed to everyone else" (p. 144).

What Drives Collective Discourse

While it is difficult to ignore the linearity (or sequencing of events) that time forces on us, we can set the boundaries of our analysis in such a way that the we change our plane of focus to that of the collective path that is laid down by the group in walking (Maturana & Varela, 1998). Here we can attend to the ideas that are bumping against each other (Davis & Simmt, 2003) rather than to the words themselves being spoken or the individuals who say them.

In determining what can help us focus on a group's ideas, we might consider what may drive utterances in the first place. Creativity researcher Sawyer notes that some artists have an improvisational style that creativity researchers call *problem-finding,* which involves "constantly searching for her or his visual problem while painting" (2000, p. 153). In their discussion of improvisational theatre, Vera and Crossan further elaborate: "As part of the creative process, actors find a problem for themselves, spend some time solving the problem, and find a new problem during the solving of the last one" (2004, p. 737). The term "problem finding" suggests that the problem exists independently of the people who find it, which belies what I believe to be the emergent nature of the process. Instead, in this paper I will use the term *problem posing* which is grounded in mathematics education literature and has been defined as "the creation of questions in a mathematical context and… the reformulation, for solution, of ill structured existing problems" (Pirie, 2002, p. 929).

In investigating the process of problem posing, Silver and Cai (2005) asked middle school students to pose three questions based on a story problem they were given. The researchers noted that the problems generated tended to be solvable (i.e., within the students' mathematical capabilities), chained (that is, produced using an associative process, in that the first problem provided a cue for the next two) and increasing in mathematical complexity (based on semantic structural relations). In their initial case study of two college-aged students each individually working on a problem posing task, Ciferelli and Cai (2005) at first suggested that the problems posed were produced in an associative manner. However, after following up with these particular students by having them work on an additional task (Cifarelli & Cai, 2006), the researchers concluded that a recursive model, where the ideas generated by the solving of one posed problem influences what problem is posed next, and so on, would be more appropriate.

Returning to Pirie's definition of problem posing, there is an issue that affects the coding of transcripts: the use of the word *question* and what it means in relation to the word *problem*. The two are often used interchangeably in everyday discussion – they frequently show up in each other's definitions – but they are not the same thing. In short: all questions contain problems, but not all problems are phrased as questions.

In everyday life, problems have a bad reputation. *Roget's Super Thesaurus* lists synonyms such as difficulty, complication, knot, trouble, dilemma, quandary, mess, pickle, predicament, can of worms, headache, pain in the neck, and hassle (McCutcheon, 1995, p. 403), all of them negative. According to *The Canadian Oxford Dictionary*, a problem is defined as "a doubtful or difficult matter requiring a solution" yet, in a mathematics context, a problem is "an inquiry starting from given conditions to investigate or demonstrate a fact, result or law" (Barber, 1998, p. 1153). Depending on one's viewpoint then, a problem in itself is not a negative thing. Still there is an element of discomfort about it, a sense that something needs to be resolved or fixed. To recognize a problem is to be aware of a gap, a disparity, a limitation, an unknown, a dissonance, a variance, a conflict, or a disconnection.

On the other hand, a question refers to the grammatical structure of an utterance, namely the interrogative form. This kind of utterance points to the existence of problem but is not the problem itself. Other language structures, not to mention physical gestures and facial expressions, can also point to problems, and this makes equating problems with questions troublesome for researchers. For this reason, I will use a revised version of Pirie's definition of problem posing (2002): "the creation of *problems* in a mathematical context and… the reformulation, for solution, of ill structured existing problems."

For the researcher, then, it is not a matter of looking for all the places in the transcript where someone happens to be asking a question. A question might point to a problem that was unrelated to the mathematical task (for instance, a student asking to drink from a peer's bottle of water), while a statement might point to a problem that formed the heart of the task. In their study of peer group discussions in elementary school classroom situations, Barnes and Todd (1995) were frustrated by their initial attempts to code the discussion by identifying questions: "We found we could not make sense of the purposes to which questions were being put if we looked at isolated cases out of context. We had to look back at what had gone before and forward to what followed" (p. 148). For instance, yes/no questions are not necessarily any more open than "wh" questions (who, what, where, when, why) – it all depends on the context in which they are posed. Ultimately, Barnes and Todd (1995) concluded that "inquiry might progress in utterances posed in any form" (p. 154) whether they be questions or statements, individually or jointly constructed.

I suggest, then, that what the researcher might look for is evidence of gaps in collective understanding that the group seemed to be actively trying to bridge (Mäkitalo, Jakobsson, & Säljö, 2009). Further, to work with the collective as the learning agent, and to focus on the level of what ideas are being developed, what must be identified are not the utterances of individual group members but utterances of the group itself. Bakhtin defined utterances as "not a conventional unit, but a real unit, clearly delimited by the change of speaking subjects" (Bakhtin, 1986, pp. 71-72), and for this study I defined a *collective utterance* as the discussion of a particular posed problem from the time it is first proposed to the introduction of the next posed problem.

Tapestry as Metaphor

To study collective discourse, we need a metaphor that will enable us to maneuver between the individual and collective planes of focus, one that offers the potential for multiple interpretations that qualitative research admits. Here, I suggest the tapestry.

Traditionally, a tapestry is made on a frame and consists of a warp and a weft. The warp provides the supporting structure, consisting of lengthwise strands, and is largely invisible to the viewer. The weft is made up of the fabrics/threads of various textures and colors that have been woven through the strands of the warp. In terms of discourse, public utterances (i.e. those that are observed) are the strands of the weft, woven together as the conversation proceeds. The warp is made up of not only "the unobservable determinants in the histories of the individuals, groups and institutional systems involved" (Mercer & Littleton, 2007, p. 121), but also the anticipation of future utterances (Bakhtin, 1981). Later in the Findings section, I will develop this metaphor further, argue that collective discourse is self-structuring, and that the warp continues to develop as a conversation proceeds as collective utterances pass from present to past, providing a supporting structure.

What is particularly helpful about the tapestry as a metaphor is its flexibility in enabling the researcher to change planes of focus. The fabric of a tapestry reveals different faces depending on its physical distance from the observer. From afar, which would be the equivalent of summarizing a group conversation and then considering it from both a temporal and contextual distance, the tapestry shows a panoramic scene – a whole composed of a number of intertwined parts. Moving closer, the landscape of the tapestry might still be evident, but now the individual strands are more visible. Moving closer still, the individual strands become the focus and the overall scene is no longer clear. In the same way, it may be easy to follow the individual turns of a conversation but difficult to summarize the gist of the discussion as a whole while it is taking place. At this close focus, the overall pattern is invisible, but individual contributions and ideas stand out. This close focus is the plane in which researchers traditionally view transcripts, but in this paper, I will take advantage of the ability of the tapestry transcript to consider discourse with a more distant vantage point.

The Study

The study took place at a grade 6-8 middle school in a large suburban school district in Western Canada[4]. Two grade 8 mathematics classes, taught by an experienced classroom teacher, Mrs. Shug[5], took part in the study, with 16 students from each class of 30 students participating in the recordings for a total of 32 students. The study was conducted in the spring of the school year, so that the social norms, values, and routines of each class had time to be established. There was a pilot taping in early March, followed by regular session tapings during April and May, roughly every two weeks depending on the school schedule. Each class had a total of five sessions, with each session lasting approximately 40 minutes.

Two stationary video cameras were each focused on a group that Mrs. Shug and I had identified as having a strong potential to work collectively with each other (which I will discuss further later) and independently from her. Also, visible in the background were other groups participating in the study, meaning that each "video-taped" group was in fact being recorded by two cameras, each with a different angle. The cameras also recorded each group participating in the study whenever it happened to be presenting its ideas to the class. There are challenges in audio-recording in a middle school classroom. Middle school classroom activities are generally noisy, particularly when there are 30 students in the room who are actively participating. As well, the video-camera's built-in microphone is often physically located too far away from the group it is recording to pick up the group's discussion consistently. To get around this, I placed an audio-recorder with each of the video groups to ensure that the group's discussion was adequately captured. In addition, I audio-recorded two additional groups per class[6] – as the workings of any group cannot be predicted, these groups served as a back-up in case they had active on-task discussions but the two videotaped groups did not.

I took field notes throughout the sessions from a location at the back of the classroom, and compared these notes to the video and audio recordings to clarify events captured in the tapings, and to make note of events occurring elsewhere in the classroom that were not captured by video. Other data sources included the task sheets where group members wrote/drew their work and solutions, and the class whiteboard where some groups chose to write/draw their ideas while presenting their solutions to the rest of the class.

As I was seeking to study groups who would work together well, Mrs. Shug and I selected students for the videotaped groups based on who Mrs. Shug thought would feel most comfortable in front of a video camera, and had the potential to actively and collaboratively engage in the assigned mathematical tasks with their peers. Thus, the groups were composed of students who were all working at grade level but who had mixed levels of ability (and confidence in their abilities) in mathematics. Some of the groups were composed of one gender,

[4] This study was authorized by the Behavioral Research Ethics Board of the University of British Columbia, certificate # H10-02716

[5] Pseudonyms were used for all participants in this study.

[6] These audiotaped groups were also visible in the background of the videos of the main groups. This allowed me to view where each audiotaped group member was positioned – and if anyone arrived or left during the session – and their gross physical movements.

while other groups were mixed, depending on the friendship groups in that particular class. We made adjustments to group composition during the study when certain students were absent, and in a few cases where the group dynamics were not working out. The groups discussed in this paper are NIJM, DATM, REGL and JJKK.[7]

The original set of tasks for the study were all from the "Problems of the Day" that I found had consistently generated on-task group discussion within my own middle years level mathematics classes. All were structured but potentially rich tasks with one correct answer and more than one possible solution path. From these Mrs. Shug chose ones with which she thought her students would be most comfortable. This paper will focus on the "Bill Nye" task[8]:

The Bill Nye Fan Club Party

> The Bill Nye Fan Club is having a year-end party, which features wearing lab coats and safety glasses, watching videos and singing loudly, and making things explode. As well, members of the club bring presents to give to the other members of the club. Every club member brings the same number of gifts to the party.
>
> If the presents are opened in 5 minute intervals, starting at 1:00 pm, the last gift will be opened starting at 5:35 pm. How many club members are there?

This task has one correct answer: there are eight club members and they each bring seven gifts. All four groups were able to reach this answer through varied solution paths.

Analysis

In considering transcript data, a researcher faces a dilemma similar to one that challenges an artist – how can she see her subject (the data) with fresh eyes? Betty Edwards, an art educator best known for her strategies for learning based on the perceptual skills of drawing, writes, "We tend to see what we expect to see or what we decide we have seen. This expectation or decision, however, often is not a conscious process" (1999, p. xxv). To get beyond these preconceived ideas, artists need to perceive their subjects differently. In the same way, a researcher needs to make her data strange in hopes of revealing new patterns and insights.

Collective Behavior

One way of distinguishing between group behaviors is in terms of how cohesively the members are behaving. When a group is acting *cooperatively*, everyone is working together to complete a task, but members of the group are focused on different parts of the task; when a group is working *collaboratively*, everyone in the group is working on the same task at the same time (Roschelle & Teasley, 1994). Finally, a group that is working *collectively* has such a high degree of coordinated interaction that it appears to be behaving as a single unit (Martin et al., 2006). The level of cohesive activity in any group necessarily waxes and wanes according to the level of members' interest and other factors, and a peak state of cohesive effort can be

[7] To reflect that the groups are made of individuals, each group's name is an acronym based on the first letters of the names of its four members. However, as the unit of study is the group, in this paper I will refer only to the groups themselves and not the individual students.
[8] I will refer to this as a "task" rather than a problem to distinguish it from the problems that the groups pose as they work on it.

difficult to sustain for long (Armstrong, 2008; Sawyer, 2003). Thus, as I wanted to document the full length of sessions where students were engaged in solving a math task, 20 – 25 minutes, I was looking for groups who were working within the range of collaborative to collective behavior. This I determined by viewing video-recordings and noting the body posture, eye contact, gestures and facial expressions of their members (Armstrong, 2008; Gordon-Calvert, 2001), and by listening to determine if all group members were discussing the task together and how receptive they were to the ideas of others. Eventually, I chose four groups who had each worked on the "Bill Nye" task. In this paper, I will refer to these groups as collectives although, strictly speaking, they are not always or only demonstrating collective behavior.

Transcribing

Any transcript represents an impossibility, as it "fixates what is essentially fluid and ephemeral" (Jordan & Henderson, p. 48), but the researcher depends upon it. Through the ability to stop and start the recordings, and to replay small clips, I was able to "improve" and expand the levels of my visual and audio attention. As Jordan and Henderson (1995) note, in transcribing "it is impossible to include all potentially relevant aspects of an interaction, so that, in practice, the transcript emerges as an iteratively modified document that increasingly reflects the categories the analyst has found relevant to her or his analysis" (p. 48). Here, I was building a document that I hoped would allow me to trace various problems posed by the groups as they worked their ways through the task.

Coding

As discussed earlier, I was looking for collective utterances, which I have defined as the posing and discussing of one problem, continuing until the next problem has been posed. The process of determining whether or not a group had posed a problem was necessarily an interpretative one. I was looking at the conversational fabric around each individual turn, both before the turn occurred and afterwards, and this involved not only reading the transcript but reading it while the video-recording and/or audio-recording were playing so that I could see/hear whether or not a problem was being taken up by the group. For example, a turn which initially appeared to be pointing out a piece of information could be treated by the group as a "What if this is true?" or a "What would happen if we try this?" type of posed problem and subsequently taken up for further discussion.

After reviewing the first group transcript, I had compiled a list of posed problems[9]. In reviewing the second transcript, I refined this list and added more problems, a process that was repeated for the third and fourth transcripts. Then, I cycled through the transcripts again, determining if any problems on the list were actually pointing to the same gap with different wording. For example, "What if there are 28 presents?" and "What if there are 16 presents?" could both be folded into a more general problem category of "What if there are *x* presents?" However, problems that at first seemed to be similar, such as "Does everyone bring the same amount of gifts?" and "Do all members give to everyone?" turned out to be pointing to different gaps of understanding.

Eventually, I had a list of 31 problems that had been posed (Table 1). I did experiment by continuing to fold these problems together until I had six different categories, and then eventually just two (problems involving interpretation and problems involving mathematical processes) but I found that the reduction in the number of categories took away from the

[9] After much thought, I decided to phrase the problem categories that emerged in the form of questions because the question is the grammatical form in the English language that is most commonly associated with problems.

richness of the tapestries. Although in the cases of both six and two categories, the resulting four tapestries were all different and provided evidence of emergent pathways, the individuality of the group's paths, showing how groups worked from various angles in interpreting a task, was stripped away. It would be like reducing the plot of a story to a short list of sentence categories that reads, "descriptive sentence," "action sentence" – one loses a sense of how the story is evolving.

Table 1: Colour Coding Chart– ordered by #s

Colour	**Problem posed (generalized)**	**JJKK**	**DATM**	**NIJM**	**REGL**	**#**
Lavender	Do we use time and divide by 5 [number of intervals]?	X	X	X	X	4
Medium blue	What about if everyone brings x gifts each?	X	X	X	X	4
Purple	Is there an extra 5 minutes? (because last gift is opened starting at 5:35)	X	X	X	X	4
Deep red	How many people are there?	X	X	X	X	4
Slate blue	What are the factors of x?	X		X	X	3
Lime green	What is meant by an interval?	X		X	X	3
Olive green	Do all members give to everyone?		X	X	X	3
Goldenrod	Do they also bring gifts for themselves?		X	X	X	3
Orange	Does everyone bring the same amount of gifts?	[X]	X	X	X	3
Sky blue	How many gifts are there?			X	X	2
Brown	What if there are x people?			X	X	2
Green	How do we think outside the box?			X	X	2
Teal	Is it a square root?	X		X		2
Fuschia	Why did we get x?	X	X			2
Dark pink	How long does it take to open all the gifts?	X	X			2
Light purple	Can they take breaks in between opening gifts?	X	X			2
Pale yellow	Does it start at one o'clock?		X	X		2
Gray	What is a tournament?				X	1
Red	What if it's an exchange?				X	1
Light green	How long does it take to open one gift?				X	1
Forest green	Can't we just count how many people?				X	1
Lilac	How many gifts does each person bring?	X				1
Coral	How many gifts are opened in an hour?	X				1
Gold	Is another group's answer right?			X		1
Sage	Can they bring partial gifts?			X		1
Pink	What if someone doesn't get a gift?			X		1
Dark blue	How do we know if we're right?		X			1
Blue	What if there are x people and gifts?		X			1
Peach	Does it take 5 minutes to open one gift or 5 minutes to open all the gifts that one person brings?		X			1
Light blue	How can we use the 24 hour clock?		X			1
Yellow	Can they open gifts at the same time?		X			1

Color-Coding

The purpose of color-coding the transcripts and turning them into tapestries was to visually highlight the emergent problem posing patterns so that they were available "at a glance" and thus provide a quick visual comparison of the four groups. The length of each collective utterance was determined by the number of individual turns in which the posed problem was discussed[10]: One line of the tapestry was assigned per contribution by a group member, regardless of the word length of this contribution. This meant that even a short "Yeah," if it was keeping the conversation going, had as much weight as a more wordy comment. There was one exception to this practice: If a single contribution contained two or more posed problems (which happened several times), then that contribution would be given a similar number of lines accordingly, with the different colors occurring in the order that their problems had been posed. Once the transcripts had been color-coded I turned each into a tapestry, which involved "shrinking" each tapestry on my computer screen to 10% of its original size and then using screen-shots to grab each image and align the four of them beside one another[11].

Discussion of Examples of Tapestries

Four groups of four grade 8 students are in a classroom mathematics class, working on the "Bill Nye" task. This is the fifth task they have worked on during the approximately two months that the study has been going on, and like the others, the task is not connected to their regular mathematics lessons, where they are currently learning about square roots. The groups have approximately 20 minutes to work on the task before the class discusses it as a whole. The four groups are all focused, and all arrive at the correct answer within the given amount of time.

What is most immediately evident in comparing the tapestries of each of the groups is the physical difference between them (Figure 1). The colors occur at different locations and in different amounts. Some colors may appear only once within a tapestry, while others appear frequently throughout. Some colors only appear in one tapestry; some appear in all. The uniqueness of each group's tapestry pattern testifies to the emergent nature of the solution paths that are developed. In this section, I will discuss some trends that may be found in these patterns, moving in to take a closer look (i.e. looking at the regular transcript) when I need more details about what is happening in the tapestry transcript.

The Role of Posed Problems

As indicated in Table 1, there are four colors that appear in all of the tapestries: lavender ("Do we use time and divide by 5?"), medium blue ("What about if everyone brings x gifts each?"), purple ("Is there an extra 5 minutes?"), and deep red ("How many people are there?"). It is tempting to consider these, as well as the problems that are posed by three groups, as being "necessary" problems, ones that must be addressed in order to complete the task. However, there are issues with this kind of generalization. First, there is the small sample size of this data.

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[10] To color code the transcript according to the length of each group member's turn (i.e. the number of words spoken), had implications. A turn by a long-winded individual group member would result in more color, even if the density of ideas in what he was saying was low. For example, he might be repeating himself, offering numerous examples of one problem, or making his point in a round-about way.

[11] Because I was working with a small number of groups, the Microsoft Office software was adequate for my coding needs. However, had I been working with a large number of groups, or sharing coding tasks with a team, it is likely that using software designed for transcript coding may have been more efficient.

Perhaps if there were more groups being analyzed, not all of them would pose these particular problems, and perhaps other problems on the list would prove to be more commonly posed.

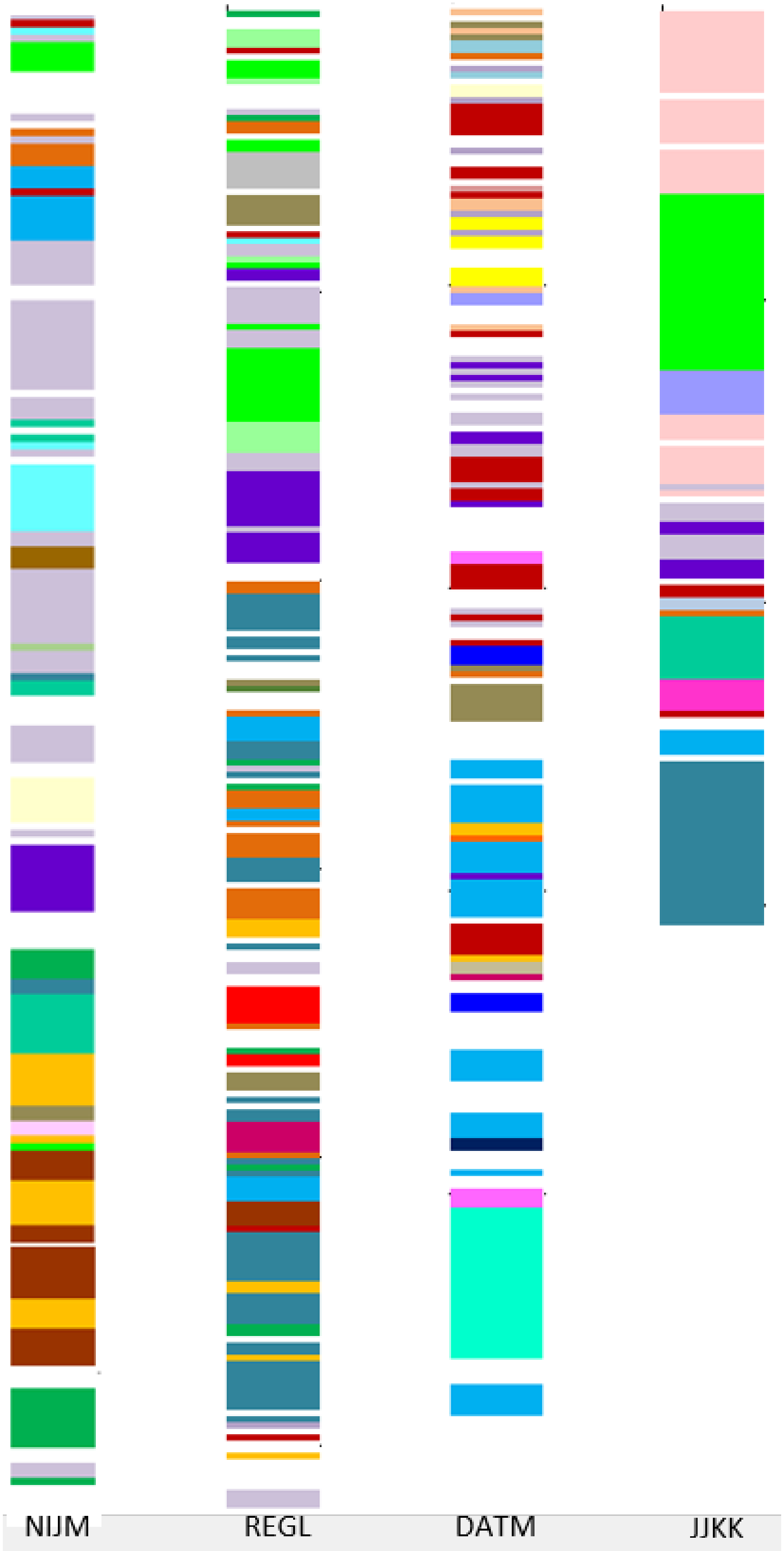

Figure 1: Tapestries

Secondly, if a group does not pose a particular problem (such as DATM not posing the "What are the factors of x?" problem that the other three groups pose), it does not necessarily follow that the group has not addressed the mathematical issue this problem touches on. It may also mean that this particular problem does not appear to be a gap in that group's collective understanding.

We cannot guess at the intent behind a group posing a problem, since we are not privy to that group's consciousness (if it exists). What we can observe is how groups take up posed problems, and it is interesting to note how the role of the posed problem changes as the group's discussion continues. For instance, on the surface, the question, "Do we use time and divide by 5?" – a problem which features predominantly in at least three of the group's discussions – may seem to be a clarification problem, but consider how it functions during NIJM's session. It is the very first problem posed by this group where it appears to be offered as a counting method. This is followed in short order by two other posed problems which seem to function as a kind of response to the task itself ("How many people are there?" "How many gifts are there?"). "Do we use time and divide by 5?" is raised a second time as the proposed counting method, and this time is explored by the group in more detail. A short break for some class discussion follows, and shortly afterwards the group poses the problem again, seeming to consider that there might be something easier the group could do than counting out the intervals in order to determine how many there are. Nothing else is suggested, and when "Do we use time and divide by 5?" is posed a fourth time, almost immediately, it prompts the counting method to begin. The fifth time the problem is posed, it is suggested that this problem will lead NIJM to determine the number of gifts each party-goer will bring. The group agrees to continue with the counting method and that if the number of intervals for one hour can be determined the group can "keep doing it" from there. The next two times the problem is raised it refers to ongoing calculations. When "Do we use time and divide by 5?" emerges for the eighth time, it is in reference to predictions the group is making as to what the final answer will be. When it occurs again, the counting is continuing. The tenth time the problem is posed, the counting has been completed and the group is considering a recount. This is followed by much discussion of other posed problems. The problem re-emerges for the eleventh, and final, time it is at the very end of the session, when the group is checking its solution, and assigning different members of the group to perform a recount. This leads to a discussion of whether or not there is another way to determine a solution. In summary, in NIJM's session "Do we use time and divide by 5?" performs the following roles:

- to propose a method of entry into the task;
- to discuss what method would be easiest;
- to discuss how it might eventually lead to solving the entire task;
- to estimate/predict possible answers;
- to narrate ongoing calculations;
- to check possible answers.

Most of the other posed problems in the study also show evidence of their roles evolving as the group discussion develops. The only time that a problem does not evolve at all is when a group does not repose it.

Unique Problems

While there are colors, such as lavender, which show up in all of the groups' tapestries, there are other colors which do not. Given that the groups are made up of students who all bring their own individual mathematical experiences and ideas to the discussions, this might be expected. What is interesting, however, is the appearance of colors representing posed problems that appear to be based on experiences known by the researcher to be shared by all of the groups. As mentioned, the groups are all studying square roots in their regular mathematics class, yet the color teal ("Is it a square root?") appears only in the tapestries of NIJM and JJKK as they consider how to determine the number of people attending the Bill Nye party, and not in the tapestries of the other two groups. In another example, groups also had experience working on a previous study task involving the use of the 24 hour clock, yet the color light blue ("How can we use the 24 hour clock?") is only found in DATM's tapestry, near the end when the group is reconsidering ways to figure out the time intervals for opening gifts at the party. This finding will not surprise any teachers who have ever thought they had successfully front-loaded students in preparation for solving specific problems, only to find that the students had taken unexpected paths and not necessarily used the information or strategies that had been rehearsed. The performance of groups is unpredictable, pointing to the emergent nature of their behavior.

Patterns

The tapestries provide visual evidence to suggest how posed problems weave in and out of group discussions. A color may appear briefly early in a conversation – for instance, slate blue in REGL ("What are the factors of x?") – and not appear again until over halfway through when it begins to occur quite frequently. A problem may be posed and seemingly disregarded by the group, only to be reposed later in the conversation. Other problems that seem to have been discussed and resolved may also reappear later for further consideration. These instances of reappearance suggest a few possibilities. As discussed above, the role of a problem changes as the path of the discourse unfolds. It may be that the problem might be considered as unimportant or uninteresting at first until the task discussion is further along and it is seen in a different context. It may be that a gap of understanding that a problem points to may seem to be resolved until further discussion opens it up again. Or it may be a matter of the group attending to other matters at first until they are ready to reconsider the posed problem. That problems so often re-emerge in the tapestries suggests the potential for problem posed early on to seed a later discussion. All of the posed problems are part of the tapestry, no matter when they are spoken, no matter how often they reappear – no utterance ever truly disappears.

Self-Structuring

As mentioned earlier, lavender ("Do we use time and divide by 5?") appears in all of the tapestries. However, it does not occur in the same locations in each of the tapestries, nor does it cover the same area. For instance, JJKK's tapestry has little lavender in comparison to the amount of coral ("How many gifts are opened in an hour?") found at the top of its tapestry and the shade of slate blue ("What are the factors of x?") that anchors the bottom. There is a lot of lavender in NIJM's tapestry, however. It appears regularly and alternates with other colors, particularly in the first half of that tapestry. It is the first problem posed by the group almost as soon as it receives the task sheet from Mrs. Shug, even before the class discussion of the task occurs, and this problem re-emerges ten more times in the course of the session. Not only does the role of the problem change, as discussed earlier, but there is evidence of the recursive nature of problem posing that Cifarelli and Cai (2006) note in their study. Many of the problems that weave in with "Do we use time and divide by 5?" appear to be generated by it, as a way of considering the issues related to this particular problem. Some of these generated problems emerge only once – such as "Does it start at 1 o'clock?" and "What if someone doesn't get a gift?" – while others re-emerge a few times. In either case, once these posed problems have been resolved to the group's satisfaction, there is a return back to the discussion of "Do we use time and divide by 5?" It seems that when "Do we use time and divide by 5?" is not visible, it seems to be acting as a kind of support, becoming part of that tapestry's warp. In that sense, posed problems might not only be considered as recursive but also as self-structuring.

Thickness of Color Bands

The four tapestries show two general patterns of emergence/re-emergence dependent on the thickness of the color bands. The thickness indicates how many turns a group takes in engaging with a problem, or the collective utterance, with slim *threads* of color suggesting a single mention of an individual posed problem, or, at most, very brief conversation about it, while *chunks* of color indicate a longer discussion.

Threads

Occasions where threads might occur include:

- situations where group members are not picking up their peers' contributions, which may occur if a member's speech is inaudible to the others, or when members are not getting along and are choosing to ignore one another;
- a group putting many problems "on the table" in order to consider what possible options are;
- a posed problem immediately triggering other posed problems to consider;
- a group juxtaposing posed problems with one another in order to develop their ideas.

To determine what is actually occurring in a particular group's discussion, the researcher needs to move from the tapestry to a closer view using the regular transcript.

There are *thready* patterns at the beginning of three of the tapestries when these groups are first considering the task. Looking more closely at REGL's transcript, the group appears to be discussing ways to interpret the meaning of the task. DATM's tapestry has a brief thready alternating pattern of lavender ("Can we take time and divide by 5?) and deep red ("How many people are there?") early in the session when the group is debating which of these two problems to pursue first. As already discussed in an earlier section, in NIJM's situation, the initial posed problem ("Can we take time and divide by 5?) appears to generate other problems for consideration. For all three of these tapestries, the threadiness near the beginning seems to indicate how each group is beginning its thinking about the Bill Nye task.

Threadiness seems to appear in tapestries anytime that a group is comparing ideas. For instance, midway through its session, REGL gets stuck. Having determined that it needs to find the factors of 56, REGL discusses all of the factor pairs except the ones that will actually lead to the final answer, 8 and 7. Realizing that something is amiss, REGL reviews the task, and in doing so it reposes most of the problems that it had discussed earlier in the session, as well as posing a few new problems along the way. This alternation of different problems results in an echo of the thready pattern evident at the beginning of REGL's tapestry when it was first generating ideas about how to approach the task.

Another tapestry location where the thready pattern is evident near the bottom of the tapestries, representing later in the sessions when the each of the three groups has come up with tentative answers and begin to repose earlier problems as a way of checking their thinking. Again, the threadiness of the pattern points to a period of comparing ideas. Even DATM, which reaches a solution just as Mrs. Shug is telling the class that it is time to wrap up their work, takes a moment or two to discuss its answer.

Chunks

When groups engage for a longer amount of time with a specific posed problem, the color bands become broader chunks. Occasions where chunks may occur include:

- situations where one group member is commanding the discussion (e.g. a talkative person, a dominant leader, someone who is passionate about a particular problem);
- when a group is debating about a single problem;
- when a group is discussing various aspects of a particular problem thoroughly in order to clarify them and to ensure that all members understand.

In this study, JJKK is a group who has a notably chunky tapestry. Moving in for a closer look at the transcript, this chunky pattern appears to reflect how a problem is posed, discussed at some length until some kind of agreement is reached, and then disappears, presumably either having been resolved or dropped completely. Take, for instance, the first problem to be posed "How many gifts are opened in an hour?" (coral). The resulting discussion explores the idea that 12 gifts would be opened in the course of an hour: this calculation is proposed as a way to begin, the group talks about where the "12" comes from until, gradually, all members of the group seem satisfied.

For approximately the first half of its session, whenever JJKK poses a problem it discusses the problem immediately and, at times, at length. Perhaps the group needs more discussion time for each problem in the beginning in order to build cohesiveness within the group in terms of how to work together and how members might interpret each other's suggestions. Given how much of JJKK's discourse appears to be required in order to establish common meanings, posing more than one problem to consider at once might be to risk confusion within the group. However, in the second half of the session, JJKK's tapestry pattern becomes less chunky, suggesting that perhaps the group members are now communicating well enough that they can assume mutual understanding of some ideas without a thorough discussion taking place first.

Further Implications of Thready Versus Chunky Patterns

While the difference between a thready tapestry section versus a chunky section is, in part, a matter of time taken with each problem, it is also matter of problems being able to "bump against" (Davis & Simmt, 2003) each other in order to make comparisons and contrasts. In order for that to occur, problems need to be reposed, and it is interesting to note the difference between JJKK and the other groups in terms of the problems it poses and reposes. In JJKK's tapestry, colors rarely repeat themselves, a result which is echoed in Table 2.

Table 2: Chart of number of problems posed and reposed by groups

Group	JJKK	DATM	NIJM	REGL
Number of unique problems posed	13	16	17	16
Total number of problems posed/reposed	23	61	45	66

While the number of different problems that JJKK poses (13) is not that much lower than the number posed by the other groups (16 or 17), the total number of problems it poses and reposes (23) is significantly lower (ranging from 45 to 66). Groups with thready tapestries tend to work with more than one problem at a time, suggesting that they are juxtaposing problems in order to negotiate meanings and new ideas, and to check possible solutions. Their discussions have the potential to be rich. On the other hand, JJKK's discussion is very linear – a problem is posed, discussed, and then the group moves on to another problem without looking back.

Discussion

The challenge this paper addresses is that of finding a way to capture the fluid and ephemeral process of group discourse in order to consider the emergent patterns of collective problem posing. What I seek is "to make visible some aspects of the dialogicality of a situation… in particular, the dynamics of collaboration over time and connect between the collective and the individual" (Grossen, 2009, p. 269). I propose the tapestry as a metaphor for documenting the emergence of collective discourse. By representing both the threads of conversation contributed by individual group members, as well as the overall gist of the conversation, represented by the patterns created by the woven threads, the tapestry enables researchers to work with various planes of focus. I see this as being a tool that researchers could use in concert with other tools in order to study collective understanding, one that allows for global analysis to identify areas of interest within the transcript for closer examination.

The metaphor also describes an analytical technique for considering collective behavior that provides visual evidence of emergent problem posing patterns. This method builds on the work of Towers and Martin (2015) by offering the potential to focus farther away in order to consider coaction from the vantage point of ideas. Posed problems are not associated with particular individuals – the dialogic standpoint belies the notion that any idea has a specific source – and instead individual speaking turns are considered evidence that the discussion is continuing, not considering who is speaking most, or the density of ideas being offering.

This metaphor does not presume the pre-existence of problem solving stages such as those that have been used in past studies to graph individual problem solving performance (e.g., Schoenfeld, 1992). Instead, the tapestry method highlights the emergent nature of collective problem posing. Although there are some problems that are posed in all groups, there are others that are not, and still others that are unique to particular groups. And the changing role of posed problems suggests that the problem solving strategies are found throughout the solving process rather than being delineated in a set of stages.

For a single task, a variety of problems may be posed by groups in a variety of patterns. Problems are posed and are reposed with the result showing how ideas are taken up and how they bump against other ideas. Looking closely enough, it is evident that there is not a single "thread" – the group's discourse is made up of utterances that weave together, weaving a fabric as they go. That problems can disappear and then reemerge points to the dialogic nature of the group discourse. Although as observers we cannot say where these ideas go and can only speculate about what they "do" when they are gone, this metaphor does speak to how collective utterances are connected to others in past, in present, and in future.

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