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Abstract

Mathematics is of utmost importance in engineering courses but studies on engineering students' conceptions of understanding in mathematics learning are found lacking in the literature. Therefore, this research attempts to address the above issue by answering the research question: "What are engineering students' conceptions of understanding in mathematics learning?" It employs the grounded theory methodology (Strauss & Corbin, 1990, 1998) and data are collected from in depth interviews with a total of 21 students and six lecturers. The substantive theory of engineering mathematics understanding (comprising of conceptual, functional, procedural, disciplinary and associational understanding) emerges in this study. The emergence of functional, disciplinary and associational understanding is unique in the context of engineering mathematics learning and has implications on successful engineering problem solving.

Keywords

Engineering Mathematics, Conceptual Understanding, Functional Understanding, Procedural Understanding, Disciplinary Understanding, Associational Understanding, Grounded Theory

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A Grounded Theory Approach: Conceptions of Understanding in Engineering Mathematics Learning

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Mathematics is of utmost importance in engineering courses but studies on engineering students' conceptions of understanding in mathematics learning are found lacking in the literature. Therefore, this research attempts to address the above issue by answering the research question: "What are engineering students' conceptions of understanding in mathematics learning?" It employs the grounded theory methodology (Strauss & Corbin, 1990, 1998) and data are collected from in depth interviews with a total of 21 students and six lecturers. The substantive theory of engineering mathematics understanding (comprising of conceptual, functional, procedural, disciplinary and associational understanding) emerges in this study. The emergence of functional, disciplinary and associational understanding is unique in the context of engineering mathematics learning and has implications on successful engineering problem solving. *Key Words: Engineering Mathematics, Conceptual Understanding, Functional Understanding, Procedural Understanding, Disciplinary Understanding, Associational Understanding, and Grounded Theory*

Introduction

Students' conceptions of understanding in mathematics can change as their learning contexts change (Kloosterman, 2002). Students may not have the same conceptions of understanding in mathematics learning when they are studying primary, secondary, or tertiary mathematics. At the tertiary level, such change of learning context might include specialization in domain areas of mathematics learning such as engineering, business or social sciences. This emphasis of the use of mathematics in specific domains is different as compared to mathematics learning at pre-tertiary levels, where a more general form of mathematics is learnt. The change of context in the uses of mathematics can possibly influence students' conceptions of understanding in mathematics learning.

In this study, the context of engineering mathematics learning in a Singapore polytechnic is set. Engineering depends significantly on the mastery of mathematics. Engineering students understand that mathematics is the foundation of engineering (Berlinghoff & Gouvea, 2004; De Lange, 1996; Graves, 2005; Peterson, 1996; Putnam, 1986; Stanic, 1989; Steen, 1988). Students' academic success in engineering courses relies greatly on their ability in mathematics as many engineering concepts could not be actualized without mathematics. Studies such as Bell (1993), Canobi (2005), Hiebert and Carpenter (1992), Mason and Spence (1999) and Yager (1991) showed that students' conceptions of understanding mathematics are important in their success in mathematics learning. However, these studies are conducted in the context of general mathematics learning and related studies on conceptions of understanding of engineering mathematics learning are found lacking. Notwithstanding that, there may be a possibility that conceptions of understanding in general mathematics and engineering mathematics may differ. Some more, if engineering students are not able to learn mathematics effectively and mathematics lecturers are not able to teach them competently because of the lack of understanding of their perspectives in studying engineering mathematics, the quality of engineers produced in the society may be compromised. Therefore, it is imperative that the issue of the conceptions of understanding in engineering mathematics learning is looked into.

To summarise, the research problem here is the lack of understanding of the students' conceptions of understanding in engineering mathematics that might give rise to the possible consequences of students' ineffective learning or/and lecturers' incompetent teaching and the eventual compromising of the quality of future engineers in Singapore. Thus, this study would centre on understanding the students' conceptions of understanding in engineering mathematics as part of their engineering diploma courses in a polytechnic in Singapore.

Purpose of the Study

The purpose of this grounded theory study is to uncover a central theory explaining conceptions of understanding in mathematics learning, as perceived by the engineering students and lecturers. In this regard, this study aims to answer the research question: "What are engineering students' conceptions of understanding in mathematics learning?" This research question would be answered in the context of the conditions below, as experienced by the participants:

1. The conceptions of understanding in engineering mathematics are understood from the perspectives of engineering students and lecturers in this study.
2. These students are currently enrolled in diploma level engineering courses in a Singapore polytechnic and the lecturers are engineering mathematics teachers.

A clear and in-depth understanding of the engineering students' conceptions of understanding in mathematics learning is important to the students themselves, their mathematics lecturers, and engineering, and other related professions reliant on mathematics. From the teaching viewpoint, the mathematics lecturers may be able to adjust their instructional strategies accordingly to ensure their students learn effectively. At the same time, the students can also alter their learning strategies usefully through understanding their own, and others' experiences in engineering mathematics learning. More importantly, it seeks to understand the study characteristics of future engineers, who form an important part of the economy. Lastly, this study contributes to the lack of literature in engineering mathematics learning.

Literature Review

Students seem to hold a variety of views of mathematics in the classroom. Across the academic levels, elementary students see that effort, regardless of ability, is the key to learning mathematics, but when they advance to high school level; they see the lack of ability as a significant impediment in mathematics learning (Kloosterman & Cougan, 1994). In the classroom, some students may believe that a good grade is important in mathematics assessment, while others may not (Hurn, 1985). Some students view mathematics learning as interesting, others may believe that it is a form of tedious and monotonous work (Cooney, 1992; Cotton, 1993). Others may even see mathematics as a subject that causes them negative emotions such as fear, anxiety and anger during lesson (Hoyles, 1982). Some students feel that they learn mathematics because of their intrinsic interest in it (Kloosterman, 2002). At the same time, some students may view mathematics learning as being forced on them by schools and teachers (Ainley, Bills, & Wilson, 2005; Cotton, 1993). These students may possibly feel that they do not understand the "purpose" of the mathematics tasks assigned to them and thus see no meaning in doing these exercises (Ainley & Pratt, 2002). Students may also feel that some of the so called real world contexts used by teachers to relate to mathematics concepts may not be interesting to them and even create confusion in their problem solving (Ainley, 2000; Silverman, Winograd, & Strohauer, 1992). In terms of learning mathematics effectively, Kloosterman reported that students view procedures as more important than concepts. They also feel that memorisation is an important part of mathematics learning (Kloosterman). In relation to social influences in mathematics learning, some students may believe that teachers make learning mathematics difficult to understand and give little guidance to their mathematics learning (Kloosterman). On the other hand, other students may view mathematics as a subject where failure to achieve the right answers is usually met with disapproval and criticism by their teachers (Ernest, 2004). At the same time, some students may recognize that peers can help or impede them in their mathematics learning (Perret-Clermont & Schubauer-Leoni, 1988; Sternberg & Wagner, 1994; Zimmer & Toma, 2000). The various studies above showed that students can hold a huge diversity of views about mathematics learning at personal, social, conceptual, procedural, cognitive and emotional levels. Such views might possibly influence how well they are able to do mathematics.

Attempts to relate the engineering mathematics learning to the above studies might not be useful for two reasons. First, most of these views are only representative of primary and secondary mathematics students and do not fully characterize those of tertiary students studying engineering mathematics. Second, all these studies are not conducted in the context of Singapore where this research is carried out. Thus, it may be shown there is a lack of such research in Singapore with regard to understanding engineering mathematics learning. This further emphasizes the importance of this study in allowing practitioners (such as engineering students, teachers, researchers and policy makers) to understand the perspectives of students in engineering mathematics learning.

Students' success in mathematics learning can be influenced at personal, social, conceptual, procedural, cognitive and emotional levels as the above studies have shown. However, this study would focus on the level of conceptual understanding in engineering mathematics learning as the meaning of understanding in general mathematics learning has been extensively discussed in the literature (Canobi, 2005; Hiebert & Carpenter, 1992; Kieran, 1992; Mason & Spence, 1999; Munakata, McClelland, Johnson, & Siegler, 1997; Rittle Johnson, Siegler, & Alibali, 2001; Sierpinska, 1994; Skemp, 1978; Stigler & Hiebert, 2004; Vincent & Stacey, 2008; Vinner, 1992; Yager, 1991). Besides, these studies also suggested possible relationships between students' understanding in mathematics learning and their level of success in mathematics learning.

In one of the earlier studies that explored the meaning of understanding in mathematics learning, Skemp (1978) first distinguished mathematics learning into conceptual/relational understanding and procedural/instrumental understanding. He believed that conceptual/relational understanding is achieved if a student comprehends the underlying principles that create a particular formula or theorem and its relationship with the other formulae or theorems. On the other hand, procedural/instrumental understanding in mathematics is achieved as long as a student can execute the rules of a particular formula in completing a mathematical task. In simpler terms, Mason and Spence (1999) distinguished conceptual and procedural understanding in mathematics as "knowing about" and "knowing to". As for Stigler and Hiebert (2004), they perceived understanding in mathematics in terms of concepts, connections and computational skills. Sierpinska (1994) suggested three ways in comprehending the meaning of understanding in mathematics – acts of understanding, the understanding that occurs from the acts of understanding and the processes of understanding. These three ways are actually processes of building, having and using mathematical knowledge respectively. Shaw and Shaw (1999) provided an interesting description of the types of engineering students learning mathematics. They classified them into five groups: Ambivalent with poor pre university teaching, Downhillers, Haters, Ambivalent with good pre university teaching and High fliers. These students perceive engineering mathematics differently in terms of its difficulty, the helpfulness of the teachers and lecturers, their level of liking and enjoyment of it, motivation in studying it and keenness in improving their mathematical capability (Shaw & Shaw). However, the study did not investigate on the types of understanding they achieve in engineering mathematics learning. A further search of research and thesis database of the only teacher training institution (National Institute of Education) and its subsidiary research centre (Centre for Research in Pedagogy and Practice) where most local educational research is done, also yields no similar studies in the local context. Since these studies are not conducted in the context of engineering mathematics learning or investigate on conceptions of understanding in engineering mathematics, this study can shed some light on how engineering students perceive understanding in mathematics.

Rittle Johnson et al. (2001) believed that a student's acquisition of conceptual understanding in mathematics can influence their level of acquisition of mathematical understanding at procedural level, or vice versa. On the other hand, Canobi (2005) also reported there is a relationship between the students' conceptual understanding of mathematics and their perceived proficiency of related mathematical procedures in problem solving. According to Vincent and Stacey (2008), having procedural competency in mathematics learning does not help to enhance conceptual understanding in learning mathematics. Kieran (1992) reported that students who failed to understand a mathematics formula conceptually would rely on memorising the procedures and thus could not see the essence of that formula. It would be interesting to investigate: If conceptual and procedural understandings do exist in engineering mathematics learning, what would be their relationship?

Yager (1991) saw the potential of improving students' conceptual understanding by teaching them using problem solving contexts. The formation of conceptual contexts in which mathematics learner can use the mathematics learned in different working contexts is mentioned in Hiebert and Carpenter (1992). If the learners are able to apply the mathematical concepts in as many as contexts as possible, their conceptual understanding of the learned mathematics would be sounder (Hiebert & Carpenter; Vinner, 1992). The above conclusion is supported by Bell (1993) who proposed that the lack of connection between conceptual and procedural understanding in terms of real world context, can be an issue in mathematics learning. Munakata et al. (1997) proposed that the ability to present concepts correctly is context dependent. The study would also help to investigate whether understanding in engineering mathematics learning is context dependent.

With respect to the fact that mathematics is indispensable in engineering education (De Lange, 1996;; Peterson, 1996; Putnam, 1986; Stanic, 1989; Steen, 1988), it would be timely to understand how engineering students perceive the concepts of understanding in mathematics learning and their possible relationships. This study can help to fill up the relevant gaps in the literature.

Role of the Researcher

Researcher's Context

It is the polytechnic context of engineering courses in which I work as a mathematics lecturer that provides the background for this research. The unique relationship between mathematics and engineering provides the foundation for this study. Mathematics is important to engineers as it is "a tree of knowledge: formulae, theorems, and results hang like ripe fruits to be plucked" (Steen, 1988, p. 611) or "a well stocked and vital warehouse" (Peterson, 1996, p. 1) where the formulae, theorems and results are at the disposal for their uses in their work solving engineering problems. As a lecturer teaching mathematics to engineering students in one of the five polytechnics in Singapore, I observe that not all engineering students do well in engineering mathematics. And the common comments given by students to explain their bad performance in engineering mathematics are related to the concept "Understanding". (Such as "I do not understand"; "I don't know how to do", "What is the use of the formulae?; "I cannot see any meaning in the formulae", etc.). Therefore, I decided to conduct this research to explore the concept of "Understanding" in engineering mathematics learning and its relationship to successful engineering mathematics learning.

Limitations

As I am a lecturer in the department in which students are the participants for this study, I am able to observe the different ways they approach engineering mathematics learning. In other words, I have prior views on the subject. I have my own perspectives about how engineering students think and feel about mathematics learning and, their subsequent behaviours towards it due to my proximity to the participants in my position as a lecturer. Such prior perspectives can bias data collection and analysis. At the same time, due to our lecturer-student relationship, some of the students are more apprehensive and reserved in sharing with me about their experiences in mathematics learning. I believe our unique lecturer-student relationship might have created a power imbalance between us. Therefore, the participants may not want to divulge honest information to me in fear of affecting their academic progress or offending me. Such power imbalance may bias the data collection and analysis stages too. In conclusion, as an insider researcher in this study, I have to acknowledge the methodological limitations caused by my preconceived conceptions about the participants and the power imbalance between us. While recognising such methodological limitations due to my proximity to the participants could not be totally eliminated, I took steps to minimise them so as to improve the rigor of the findings.

Insider research is very useful for teachers in exploring and developing their own practices in mathematics education (Jaworski, 2004; Jaworski & Goodchild, 2006). However, there are some research limitations that come with it. These limitations need to be tackled effectively to improve the rigor of this study. The first important limitation of insider research that was addressed here is the pre-understanding/familiarity of the researcher in the research setting (Brannick & Coghlan, 2007; Mercer, 2007). As I have been teaching in this polytechnic for a number of years, I have experience in dealing with different types of mathematics students and their behaviours. This is beneficial as it makes me more theoretically sensitive and I am also able to elucidate the meanings of data analysed more effectively. However, such pre- understanding/familiarity can create preconceptions about the researched phenomenon and this can bias the study (Brannick & Coghlan; Mercer). Such bias can be created in the data collection and analysis stages through the researcher's failure to probe areas outside their scope of research knowledge, unsubstantiated assumptions of the types of data to be collected and unwillingness to reframe their current thinking in data analysis (Brannick & Coghlan; Mercer).

To reduce such limitation, I was consciously aware of my beliefs, values and possible biases due to my familiarity with, and experiences in, the research context in the data collection and analysis processes by recording them in my memos. By being constantly reflexive, I maintained a self awareness of such pre-understanding bias by continually appraising the subjective responses and intersubjective relationships within the data analysed in relation to my potentially biasing experience (Brannick & Coghlan, 2007; Finlay, 2002; Nightingale & Cromby, 1999).

The next limitation discussed here is the power imbalance between the insider researcher and the participants. As a lecturer in the same polytechnic as the students, I have ready access to potential student participants. Besides, I am also able to build rapport with them more easily (Mercer, 2007). However, disadvantages in terms of power and trust relations between the participants and the researcher may be created as a result of such proximity (Mercer; Wengraf, 2001). Power imbalance between them may result in the reluctance of the participants to divulge important information or to question the validity of the research and its analytic framework and results (Hall & Callery, 2001). This is because they may not want to offend the researcher and/or affect their academic progress due to the researcher's higher authority in academic settings. Thus, the participants may not trust the researcher with information that they feel may disadvantage them. As a result, some participants may not want to divulge information that is against the social norms and may instead give socially desirable responses. This might well greatly affect the rigor of the research. Therefore, to remove such limitations caused by power and trust relations, I ensured that the participants were not my former students, nor would they be my future students. This helped prevent any bias due to role duality (the situation where I teach and research on the same set of students) in the research process (Brannick & Coghlan, 2007). At the same time, I shared my personal and professional values with the participants so as to gain their trust, which in turn, enabled more honest disclosures from them. They were also assured that the research would not affect them academically or personally in any way. Such honest communication is known as the processes of relationality and reciprocity (Mercer; Hall & Callery).

Methodology

The interpretive research paradigm can answer the research question posed in this study more appropriately as compared to the positivist research paradigm as the questions aim to understand the students' perspectives of studying engineering mathematics within a context instead of seeking causal relationships among variables related to it (Cohen, Manion, & Morrison, 2000; Donmoyer, 2001; Shaughnessy & Zechmeister, 1997). Besides, few if any study of students' perspectives of studying engineering mathematics has been conducted in Singapore and the inductive nature of the interpretive paradigm is best suited for such studies with no or few existing theories (Cohen et al.; Nassar, 2001).

There is a wide selection of approaches in the interpretive research paradigm to use in this study (such as grounded theory, ethnomethodology, life history, inductive analysis, phenomethodology and case study). In the selection of any research methodology in a study, Field and Morse (1985) highlighted the importance of its methodological relevance to the research problem and the congruence of the researcher's epistemological focus with it. On the other hand, Nassar (2001) emphasised the type of training a researcher has and his/her research experience also influence the choice of the research methodology in a study.

This study attempts to investigate the experiences of students studying engineering mathematics through understanding their thoughts, feelings and actions/interactions. Therefore, the symbolic interactionist aspect of grounded theory which assumes that one's communications and behaviours express meanings as influenced by social conditions and interactions around them, suits it well (Blumer, 1969; Grbich, 1999; Mead, 1964). In short, grounded theory allows processes, meanings and relationships in the phenomenon of studying engineering mathematics to be investigated. On the other hand, there are few if any theories that are associated with the students' experiences of learning engineering mathematics. As grounded theory is known for building theories inductively from data in under-researched areas, it is suitable for the exploratory nature of this study (Grbich). Besides, as a researcher who prefers to work within clearly determined processes, grounded theory suits me as it is more rigorously systematic in its data collection and analytical procedures. In addition, I have always been more familiar and confident with grounded theory methodology due to my postgraduate training in it. Thus, in line with Field and Morse (1985) and Nassar (2001), I have chosen to use the grounded theory methodology in this study.

As grounded theory is popularised, differences in its methodological assumptions and methods arise. There are different variations of grounded theory methodology 40 years after its birth (Mills, Bonner, & Francis, 2006). According to Creswell (1998), there are three main groups of grounded theorists – Glaser (1992, 1998), Strauss and Corbin (1990, 1998) and the social constructivists such as Charmaz (1990, 2006) and Annells (1996). However, it is not feasible to discuss their differences here due to space constraint. Nevertheless, notwithstanding the differences in the methodological assumptions and methods among the different schools of grounded theory, its philosophical roots and main canons remain the same (Creswell, 1998). In this study, I used the methods proposed by Strauss and Corbin (1990, 1998) because its detailed and systematic methods are more user friendly to me.

Ethical Issues

Informed process consent, confidentiality and emotional protection were ethical issues addressed in this research (Kvale, 1996).

Informed consent. Before the start of the interview process, there is a need to let the participants know the purposes and implications of the research (Bogdan & Biklen, 2003; Kvale, 1996). Therefore from there, it is only possible to get the informed consent of the participants. Evans and Jakupec (1996) and Kvale perceived informed consent as one of the most critical issues in interpretive research although there are others such as Fine (1992) who felt that covert research is acceptable as long as the participants are not harmed. I do not agree with Fine as it is the rights of the participants to be in the know. There is a need to confirm informed consent during the different stages of the research (Bartunek & Louis, 1996). As the events unfold during different stages of research, the participants need to be consulted for consent if it is deemed that any new development may harm them. Lastly, participants must be given ample opportunities to withdraw from the research proper without any explanation.

In this study, the participants were informed of the research's purposes and implications. They were then asked to consent to the study without coercion. They were given one week to consider the request. During the interview process, they were allowed to terminate it anytime without reasons. At the same time, I ensured that the content and analysis of the interview transcript of a participant go through him/her for checking before being released to others. If the participant did not agree to release the analysis for any reason (his/her reason would not be asked), his/her decision would be respected unconditionally. In this study, no participants terminated the interviews halfway or objected to my analysis in the report.

Confidentiality. The confidentiality of the participants must always be protected (Bogdan & Biklen, 2003; Fraenkel, 1990; Kvale, 1996). In this study, no participant was identified without their approval. The interview transcripts were returned to the participants and the subsequent analysis report were sent to them for review in case that any part of the analysis might identify them (Rowling, 1994). This protects the participants from any form of embarrassment or harm (Bogdan & Biklen). However, Berg (1998) stated that maintaining the anonymity of the participants may not be able to protect the confidentiality of their identities if other information provided may identify them indirectly. Therefore, I took care in not revealing other information that may divulge their identities in my report. In this aspect, I had given pseudonyms to the participants' school, courses, names and gender. Their tutors were not identified too.

Emotional protection. The researcher has to be aware about the issue of sensitivity when dealing with emotional research questions (Bogdan & Biklen, 2003). In short, the interview process must not cause any emotional distress to the participants. The researcher must actively watch out for signs of such emotional distress and terminate the interview when necessary. The nature of this study will not cause any physical harm to the participants directly from the interview process or the release of the research findings. However, it may elicit memory of bad experiences of mathematics learning. Therefore, in this study, I constantly watched for such emotional distress from the participants and would terminate the interview sessions when necessary. However, there was no instance where I needed to terminate the session due to the interviewee's emotional stress.

Context of Study

In the case Polytechnic, all students, most of whom, are aged between 17 and 20 years old, undergo two years (two semesters per year) of compulsory engineering mathematics learning (comprising of four modules) as part of their engineering courses. They learn mathematics concepts that include algebra, trigonometry, complex numbers, calculus, Laplace transforms, statistics, methods of integration, differential equations, infinite series and Fourier series. Then the students need to apply all these mathematics concepts extensively in their other compulsory engineering modules required in the completion of their engineering diploma courses. At the same time, these mathematics concepts tend to be more difficult for the students to master than those they learnt in secondary mathematics education, due to their mathematical complexity and relationships to engineering. Also, the focus of engineering mathematics education centres on skills acquisition in relation to engineering theories. Such a unilateral approach to the engineering mathematics curricula can influence the students' perspectives of engineering mathematics learning.

Data Collection Methods

This section will detail the various stages of data collection in this study.

Setting and participants. This research was conducted in a polytechnic in Singapore. As there are many types of engineering courses (with different mathematics syllabi) in this polytechnic, any attempt to investigate all courses is futile due to the possible extensive range of experiences that cannot be adequately covered within the confines of time and space of this study. Thus, the sample for this study was drawn from selected courses in the School of Electrical and Electronics Engineering. The sampled students in the four diploma courses – Electrical Engineering, Aerospace Engineering, Bioelectronics Engineering and Communication Engineering, needed to take a centralised programme (that consists of three modules) in engineering mathematics. Although the students' experiences of mathematics learning in other engineering disciplines are not covered in this study, they should serve as possible areas of inquiry in forming more substantive theories in future.

Potential participants were first briefed on the purpose of the study - to understand their experiences of learning engineering mathematics. They were also told that the consequences of the research could improve teaching and learning of engineering mathematics and at no time, their grades or performance in their study here would be affected by it. They were then informed that the interviews would be confidential and they were allowed to discontinue at any point of the interview without giving any reason. These measures thus ensure the ethics of interview research (informed consent, confidentiality and its consequences on the participants) are adhered to (Kvale, 1996). Each participant was given a week to consider if he/she wanted to take part in this study. A total of 24 students were invited to participate in this study and 21 students agreed to take part. The three students who did not wish to participate in the study cited that they did not have the time for the interviews.

The interviews of these 21 students served as the main source of data for this study. Data were collected through interviews that centred on the four fragmented specific research questions for this study. These questions were based on the concepts derived from the literature review and my personal experience relevant to the phenomenon studied. However, as these concepts did not evolve from the data, they were considered as provisional and could be discarded if they were not theoretical relevant eventually (Strauss & Corbin, 1998). During the initial open sampling stage, a semi structured interview guide that focused on the research question was used. The initial interview guide changed four times for the purpose of theoretical sampling (Strauss & Corbin, 1998).

All interviews were conducted in the privacy of soundproof classrooms or library special rooms. Throughout the interviews, I conscientiously attempted to improve my interviewing skills by following the desirable qualities (knowledgeable, structuring, clear, gentle, sensitive, open, steering, critical, remembering and interpreting) of an effective interviewer as suggested by Kvale (1996). A digital voice recorder was used in recording each interview. Each interview lasted from 15 minutes to 35 minutes. There was a total of 499 minutes of interviews in all. All interviews were recorded, transcribed verbatim and stored in a computer database. There were also five follow up interviews which were done through email correspondence. They were conducted to clarify and validate certain categories and their relationships. Email correspondence was used as it was considered more efficient than fixing another face to face interview with them.

Another source of data was obtained through the informal conversational sessions (each lasting less than ten minutes) with the 21 interviewed students' tutors. Two tutors who were approached in the study did not agree to be involved in the sessions. I did not ask them their reasons of declining the interviews. In the end, a total of six tutors (three of them were the ones who recommended me the interviewees) were involved in these sessions. Some of them were either the current or former tutors to a few of the interviewed students. Questions asked in such sessions were essentially related to their perceptions of their students' attitudes in engineering mathematics learning. These sessions were not audio-taped as four of them were uncomfortable about the recording but notes were taken during the interviews. However, this should not affect the rigor of this study as the information obtained was considered complementary since this study focuses primarily on the perspectives of the students. Thus, this source of data from the tutors was mainly used to triangulate the main sources of data collected through the students' interviews.

Sampling method. In grounded theory, sampling is conducted according to the principle of theoretical sampling where sampling choices are dictated by the categories of the emerging theory. In theoretical sampling, the researcher jointly collects codes and analyses his/her data. He/she then further decides what the data to be collected next are and where to find them, so as to develop the theory as it emerges (Glaser & Strauss, 1967; Strauss & Corbin, 1998). Thus, the function of theoretical sampling is to aid in the selection of participants who will yield data that produce categories of a phenomenon until no new categories are found. At the same time, it also helps to develop, elaborate and refine existing categories through searching their other uncovered properties and dimensions, until none is found. In other words, the researcher is "sampling along the lines of properties and dimensions, varying the conditions" (Strauss & Corbin, 1998, p. 202). Therefore, theoretical sampling is not searching for repeatedly occurring properties or dimensions of a category (Charmaz, 2006). In short, theoretical sampling increases the range of variation and analytic density of the categories and puts them on firmer and stronger theoretical grounds (Strauss & Corbin, 1998). In this study, open sampling was utilised (Strauss & Corbin, 1998).

Open sampling. At the start of any study, open sampling has to be employed as no theoretically relevant categories are as yet uncovered (Strauss & Corbin, 1998). Open sampling aims to discover initially emerging theoretically relevant categories that may serve as the basis for theoretical sampling later. In open sampling, the participants are selected based on their expert knowledge of the phenomenon to be studied, that is, students who can share information about their perspectives of studying engineering mathematics in this study. There are different variations in open sampling (Strauss & Corbin, 1998). In this study, the initial open sampling of engineering students was based on the literature review and my professional experience. I observed that proficiency in mathematics can significantly determine the types of experiences of mathematics learning. Proficiency in mathematics is especially relevant in the Singaporean context where mathematics education is a highly competitive and stressful journey which affects the students academically, economically and socially in their future lives (Fan, Quek, Zhu, Yeo, Lionel, & Lee, 2005; TIMMS, 2003).

I involved three of my colleagues in this study and enlisted their help in recommending students who satisfied the initial sampling criteria. Thus, the selected students were not my current or former students. At the same time, I would not have a chance to teach them as they had either completed their three compulsory mathematics modules recently or were doing their last mathematics module when I interviewed them. This greatly reduces the power imbalance between the interviewer and interviewee which can affect the trustworthiness of the study (Wengraf, 2001). This should also enable them to speak openly and honestly without the fear of affecting their current or future grades, or offending me. In this study, eight students were initially selected according to their differences in academic abilities in mathematics (good [A-C grades] and mediocre [D-F grades] in their past engineering mathematics modules) across the four diploma courses. This enabled maximum variation in the initial open sampling in this study.

Before the next round of interviews was carried out, data transcription and analysis were completed for the previous interviews. This is to prevent the researcher from missing the chance to sample and collect more information regarding new emerging concepts (Strauss & Corbin, 1998). There were two more groups of interviewees that consisted of eight and five students respectively.

Data Analysis Methods

The eventual generation of original, and yet justifiable, theories depends on the rigorous operationalisation of the data analysis phase in grounded theory (Strauss & Corbin, 1998). The data analysis phase is the stage where the data are broken down, conceptualized and creatively put back in new ways. From there, new concepts are built from the data that contribute to the creation of new theory or theories. Data analysis is achieved through the process of coding as dictated in grounded theory methodology (Charmaz, 2006; Glaser, 1992; Strauss & Corbin, 1998). Grounded theory method relies on the method of constant comparison where the analysis of data involves the data interacting with one another through comparison.

Constant comparative method. In the constant comparative method, each piece of relevant data is continually compared with every other piece of relevant data to generate theoretical concepts that encompass as much behavioral variation as possible (Glaser & Strauss, 1967). It involves four stages: (a) comparing incidents applicable to each category; (b) integrating categories and their properties; (c) delimiting the theory and (d) writing the theory (Glaser & Strauss, p. 105). It is important to compare every incident and category with one another (Strauss & Corbin, 1998). This is done through asking questions of the information provided by each incident or category to identify whether any two are similar. Through this comparison process, the collection, coding, and analysis phases work in tandem from the start to the end of the investigation. This allows the gradual development of the data from the lowest level of abstraction to a higher one of theoretical conception (Strauss & Corbin, 1998). At the same time, theoretical sensitivity, which is important in the data analysis stage, is fostered in the constant comparison phase (Strauss & Corbin, 1998).

Theoretical sensitivity. Theoretical sensitivity is very important in the analysis stages of grounded theory methodology. Theoretical sensitivity is the ability of the researcher to identify the important features of the collected data, perceive the concepts, categories, properties and their interrelationships that arise and finally give meanings to them (Glaser, 1992; Glaser & Strauss, 1967; Strauss & Corbin, 1998). In the initial stage of data analysis, certain events may be overlooked, but as theoretical sensitivity increases, they can be recoded and reanalyzed (Strauss & Corbin, 1998). Glaser and Strauss suggest that the researcher's personal inclinations and experience are helpful in creating theoretical sensitivity to the ongoing research. In this study, I was able to draw on my former experiences as a secondary school mathematics teacher and in my current appointment as a polytechnic lecturer, teaching engineering mathematics. This helped me to attain an acceptable level of theoretical sensitivity in dealing with the initial data collection and analysis. At the same time, the reading of literature also helps in enhancing the theoretical sensitivity of the researcher as it exposes him/her to the different dimensions of a phenomenon (Glaser, 1978). Thus, the literature review conducted was very helpful in developing my theoretical sensitivity. As the study progressed, the data analysis phase became another source for increasing my theoretical sensitivity due to my exposure to more aspects of the investigated phenomenon (Strauss & Corbin, 1998).

Although it is recognized that pre-existing perspectives and knowledge can develop theoretical sensitivity to enhance theory development, their meanings may also be forced into the data and limit theory development (Charmaz, 2006; Glaser, 1978). This is because a researcher's background perspectives and knowledge play the role of "points of departure" (Charmaz, 2006, p. 17) where they set the grounds for asking interview questions, the way that the data are collected, organized and analyzed. Because of the presence of pre-existing conceptions from his/her background perspectives and knowledge, certain data may be filtered out consciously or subconsciously. Then the researcher will not be able to identify some of salient features of the data. This can result in an inaccurate portrayal of the phenomenon investigated.

To prevent any filtering of data through such pre-existing conceptions, I have constantly evaluated the fit between my preconceptions and the emerging data by looking at data from multiple angles, making constant comparisons, going towards any possible new directions and building on any feasible ideas (Charmaz, 2006). This is also known as the process of theorizing (Strauss & Corbin, 1998).

Open coding. In this study, three types of coding are utilized: open coding, axial and selective coding (Strauss & Corbin, 1998). Open coding involves the labelling and categorization of the phenomenon as indicated by the data. Coding does not entail the mini-descriptions of the different blocks of data but it works in capturing the meanings of theirs instead (Charmaz, 2006; Strauss & Corbin, 1998). The end products are concepts which are the building blocks that will help build up the grounded theory. The comparative method that employs the procedures of asking questions and making comparisons is being utilized in the open coding process (Glaser, 1978). By asking simple questions such as who, why, what, how, when, where etc, every word, phrase, or sentence in each line of data is analyzed. Each analyzed line is then broken down microscopically into different discrete events called codes. All codes are assigned individual incident labels. Such line by line microscopic analysis serves to prevent researcher from making biased analyses due to preconceived ideas about the data or theory as microanalysis forces the researcher to be exposed to the complete range of the data (Strauss & Corbin, 1998). Open coding then progresses to the platform where the codes are compared and similar codes expressing the same incidents are grouped together under the same conceptual label. Each such group thus becomes a concept. These conceptual labels are then contrasted again and further clustered into a higher and more abstract level known as categories (Strauss & Corbin, 1998). After a category is formed, there is a need to identify its properties and dimensions (Strauss & Corbin, 1998). The properties of a category can be obtained from the same principle of asking who, why, what, how, when, where questions within the category. The dimensions of the category include the range of situations with such properties have occurred for the subjects. Examples of opening coding where the eventual category of associational understanding is first named under different codes, are shown below in Table 1:

Table 1. Examples of open coding

Interviewee	Transcripts	Open Coding
E01	Somehow, it does lah. After all, in electronics, it still covers things like complex numbers and it is already covered before mathematics. So that we have a more clear picture of what we studied in electronics, sine waves, cosine waves, all these kinds.	*Acknowledging the importance of EM; Understanding the Relations of maths concepts in engineering concepts*
E05	Newton's Law lah, Ohm's Law lah, and things, and things like, and things like, laws came in you need to apply mathematics into, into it lor, when you calculate formulas lah, or we, we found out what's the current, things like that. We actually use Mathematics to solve, circuit problems lor… Ah.	*Physics related to maths formulae*
E06	Ah… It is link in a way that. Ah… Sometimes when, the lecturers teach us Engineering Mathematics… Link some of the…, some of the cases the problem to, to eng-, to the engineering right, like how you analysis the circuit.	*EM related to engineering concepts*

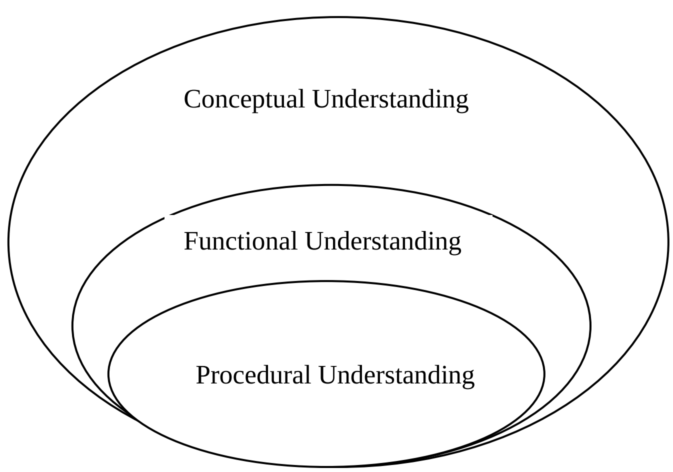
Axial coding. As for axial coding, those assembled data are put back together in fresh ways by making associations between a category and its subcategories (Strauss & Corbin, 1998). This is to bring together the categories and subcategories in explaining the phenomenon that is embedded in the data. The development of main categories and subcategories is central to the process. The first step in axial coding is the identification of the properties and dimensions of each higher order category or subcategory as compared to those in open coding (Strauss & Corbin, 1998). The second stage involves the exploration of the relationships between them and uncovering the conditions, actions and consequences for the phenomenon through these relationships. Using the same examples in open coding, the process of axial coding is demonstrated below in Table 2.

Selective coding. Selective coding refers to the integration of the categories to structure the initial theoretical framework so as to analytically come up with the grounded theory from the data. The first step involves the identification of the core category. Strauss and Corbin (1998, p. 146) stated that the core category "is a conceptual idea under which all the other categories can be subsumed." The core category is, in fact, the conceptualization of the storyline about the central phenomenon of the research study. The core category is the main theme of the data such that it can explain the whole phenomenon investigated. It is also important that the other categories must be able to relate to the core category in the description or explanation of the whole phenomenon. Therefore, the auxiliary categories may be linked to the core category in complex and intertwining ways. There is a need to note that there may be more than one core category that represents the phenomenon investigated. At the same time, the data form each category must not be forced into forming a relationship with the core category/categories. There are four methods to identify a core category, namely writing a storyline, conceptualization, use of diagram and review of memos (Strauss & Corbin, 1998). An example of selective coding in the use of diagram is shown below in Figure 1.

Table 2. Example of axial coding

Open Coding	Axial Coding		
	Category Name	Properties Mathematical knowledge Engineering concepts (Physics) Relating	Dimensions Types of formulae Level of understanding of formulae Types of engineering concepts Level of understanding of engineering concepts Types of relationship of formulae to engineering concepts. Level of understanding of formulae and engineering concepts Level of exposure to engineering related mathematics problems
Acknowledging the importance of EM; Understanding the Relations of maths concepts in engineering concepts Physics related to maths formulae EM related to engineering concepts	Associational Understanding	Context Lectures, tutorial, self studying	Intervening conditions Lecturers, peers, self
		Action / interaction strategies Students aim to achieve the different levels of associational understanding through different strategies.	Consequences They may achieve different levels of associational understanding.

Figure 1. An example of selective coding



Rigor

According to Lincoln and Guba (1985), the basic question addressed by the notion of rigor in interpretive research is "How can an inquirer persuade his/her audiences that the research findings of an inquiry are worth paying attention to?" (p. 290). The rigor of an interpretive study is examined by the notion of trustworthiness. Trustworthiness is defined as the conceptual soundness of the research results and is influenced by the notions of credibility, transferability, dependability and confirmability (Lincoln & Guba).

Strauss and Corbin (1998) set the criteria for judging the rigor of a grounded theory as plausibility, reproducibility, generalizability, concept generation, systematic conceptual relationships, density, variation, and the presence of process and broader conditions. These criteria translate into eight conceptual questions to judge the trustworthiness of the study (Strauss & Corbin, 1998, pp. 270-272):

1. Are concepts generated?
2. Are concepts systematically related?
3. Are there many conceptual linkages and are the categories well developed? Do categories have conceptual density?
4. Is variation within the phenomena built into the theory?
5. Are the conditions under which variation can be found built into the study and explained?
6. Has process been taken into account?
7. Do the theoretical findings seems significant and to what extent?
8. Does the theory stand the test of time and become part of the discussions and ideas exchanged among relevant social and professional group?

These criteria in determining the rigor of a grounded theory can be complemented by ensuring the credibility, transferability, dependability and confirmability of the study are satisfied (Guba & Lincoln, 1989).

Credibility. In terms of interpretive inquiry, internal validity is almost impossible to achieve (Lincoln & Guba, 1985). This is because one needs to know the one objective reality that represents the phenomenon that is investigated, which is unattainable in interpretive research. Therefore, an interpretive study will not set to prove if this represented reality of the phenomenon is the one objective truth in positivist tradition. Instead, it should strive to describe accurately the reality of a phenomenon it intends to represent within the research context. This is thus known as the credibility of the study.

Theory/perspective triangulation in this study was achieved through the literature comparison stage as proposed by Glaser (1992). By tying the emergent theory with extant literature after the analysis, the credibility of the theory building can be enhanced (Glaser, 1992). Triangulation of sources was achieved here through ensuring maximum variation in the sampling of interviewees and types of data sources (Hammersley & Atkinson, 1995; Patton, 1990). This also improves the credibility of the study.

Transferability. Transferability refers to the applicability of the research findings to other similar settings (Lincoln & Guba, 1985; Merriam, 2002). In this study, a reader can achieve the transferability of the data through reading the thick descriptions (Geertz, 1973) of how engineering students perceived understanding in mathematics learning in the context of a polytechnic (Lincoln & Guba). This will allow others to make accurate and informed judgements if the findings of this study are applicable to their contexts.

Dependability. Dependability refers to the consistency between the data collected and the findings (Lincoln & Guba, 1985; Merriam, 2002). An audit trail that consisted of a detailed documentation of the methods and the collection and analysis of data was maintained to ensure the dependability of this study (Merriam; Seale, 1999). This audit trail included the list of interviewees, interview guide, audio records and transcripts of interviews, data collection and analysis procedures, memos and results.

Confirmability. Confirmability refers to the degree the findings can be corroborated by other researchers (Merriam, 2002). A researcher can improve confirmability of a study by being reflexive (Merriam). Reflexivity is a process of conscious self awareness where a researcher continually appraises the subjective responses and intersubjective relationships within the data in relation to his/her values, experiences, interests and beliefs (Finlay, 2002; Nightingale & Cromby, 1999). My reflexivity was maintained by being consciously aware of my epistemological preferences, beliefs, values, theoretical orientations, bias, experiences and recording them in my memos. In addition, the audit trail mentioned above helped in enhancing the confirmability of this study.

Findings

Before the various types of understanding are discussed, the meaning of theorems, formulae and procedures unique in engineering mathematics will be defined. Mathematical theorems are statements that can be proven on the basis of agreed mathematical assumptions and can be represented in the form of formulae. In engineering mathematics, theorems are usually represented as formulae. Procedures refer to the steps in operationalizing the theorems or formulae. In this study, it is discovered that understanding in engineering mathematics learning can be conceptual, functional, procedural, disciplinary and associational.

Conceptual Understanding

Conceptual understanding refers to the participants' ability to understand how the mathematical formulae are derived. In other words, it indicates the basis of the mathematical assumptions and proofs of the mathematical formulae. The students' comments below describe the notion of conceptual understanding when they were asked about their perceptions of the formula, $\frac{d}{dx}\left[x^5\right]=5x^4$:

> It is good to know the theories or proof behind the formula and why it is like this. (E06)

> If I know why this formula comes about [the proof] why it is put this way, I can understand it better. (E08)

T01, T02, T03 and T06 agreed that their students could not understand how the formulae are derived although the lecturers demonstrated their proofs.

Functional Understanding

Functional understanding refers to the participants' actual comprehension of the functions of the mathematical formulae and procedures in the mathematics domain. The participants' remarks below demonstrate the meaning of functional understanding when they were probed about what they think of the formulae, $\frac{d}{dx}\left[x^5\right]=5x^4$ and $\int\left[\cos x\right]dx=\sin x$ respectively:

> …it shows how much x^5 changes when x is changing every second. (E13)

> In maths, this also means we find the area under cos x, if two points are given. (E06)

> In mathematical terms, some of the students do understand the use of these mathematical formulae; for example, they know that a definite integral allows them to find the area under the curve. (T04)

Procedural Understanding

As perceived by the participants, procedural understanding refers to the ability to model the steps in the mathematical formulae in solving mathematical problems. The category of procedural understanding is shown by the participants' comments below with regards to the formulae, $\frac{d}{dx}\left[x^5\right]=5x^4$ and $1+j=\sqrt{2}\left[\cos(\frac{\pi}{4})+j\sin(\frac{\pi}{4})\right]$ respectively:

> Yes, this is the differentiate x^n thing, if I see it in exams, I just need to put in the formula and fill up the values, I think… (E07)

> To get the answer (Right hand side) just got to key the values (Left hand side) into the calculator. (E19)

All six tutors unanimously concurred that their students are proficient in modeling steps of formulae in solving mathematics questions.

Disciplinary Understanding

Disciplinary understanding refers to the students' ability to understand engineering concepts and theories in their respective disciplines in engineering they are enrolled in. As engineering students, they need to know the various engineering concepts and theories in solving engineering problems. From the data, it is perceived that such disciplinary understanding might be important in engineering mathematics learning (it will be elaborated in the latter sections). Two illustrations are given below.

> I have to know the concept of electrical circuits in learning engineering mathematics too. (E01)

> I always remind my students that engineering mathematics also entails them understanding physics concepts and they also acknowledge it too. (T03)

Associational Understanding

Associational understanding refers to the participants' ability to relate and utilize the mathematical formulae in the engineering problems they are tackling. This means that the participants convert mathematical formulae into engineering terms and apply the formulae in solving engineering problems. Associational understanding is mentioned in the participants' accounts below with regard to the equations of $y = 2\sin\left[x + \frac{\pi}{3}\right]$ and $\frac{d}{dx}\left[x^5\right] = 5x^4$ respectively:

> We make use of the sine wave to calculate our AC curve, AC power supply, the components that make the AC supply. (E08)

> …(the formula) help you calculate in the design of circuit or stuff like that.… (E11)

> To be able to understand which formulae to use in solving engineering problem is important. (E06)

Relationship among the Five Types of Understanding

Relationships among conceptual, functional and procedural understanding. From the analysis of the students' own perceived levels of accomplishment of the different forms of understanding and the tutors' feedback, the relationship between conceptual, functional and procedural understanding is further conceptualized. Participants who have achieved conceptual understanding can naturally attain both functional and procedural understanding. Below are the comments of *T02* and *E21* that supported the conclusion above.

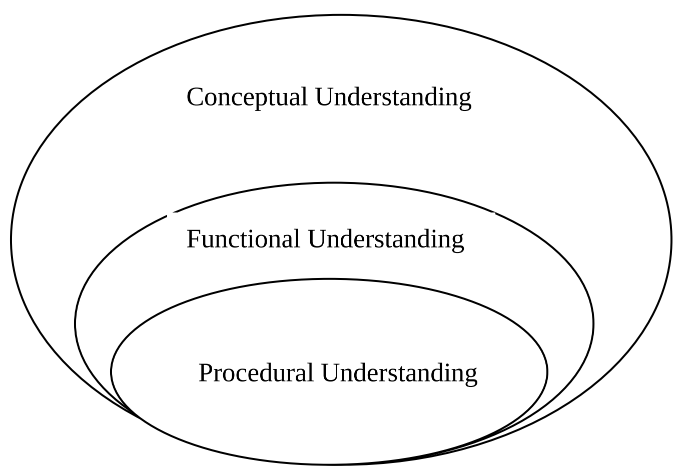
> My observations of my students throughout these years tell me that those who understand the proof [conceptual understanding] behind the formulae are always able to know the contexts they [formulae] work in. And for the steps of the formulae [procedural understanding], they have totally no problem in it. But I cannot say for students who cannot grasp the proofs of the formulae.
>
> As long as I make sure I know the proofs of the formulae, I usually have no problem with its uses and steps. (E21)

On the other hand, participants who have attained functional or procedural understanding might not be able to reach conceptual understanding as they do not understand explicit assumptions and proofs behind the formulae. Participants who have accomplished functional understanding are more likely achieve procedural understanding. However, participants who have only achieved procedural understanding usually fail to accomplish conceptual or functional understanding as they do not understand the circumstances a formula is created and used. The comments by E16 and T06 below illustrated the above conclusion.

> I do not see the need for me to know the proofs of the formulae as I still can easily understand and carry out their steps [functional and procedural understanding]. (E16)
>
> During lessons, I see that students who find it difficult to see the proofs behind the mathematics formulae still can compute their steps without mistakes. (T06)

The diagram below illustrates the relationship among conceptual, functional and procedural understanding.

Figure 2. Conceptual understanding in relation to functional and procedural understanding



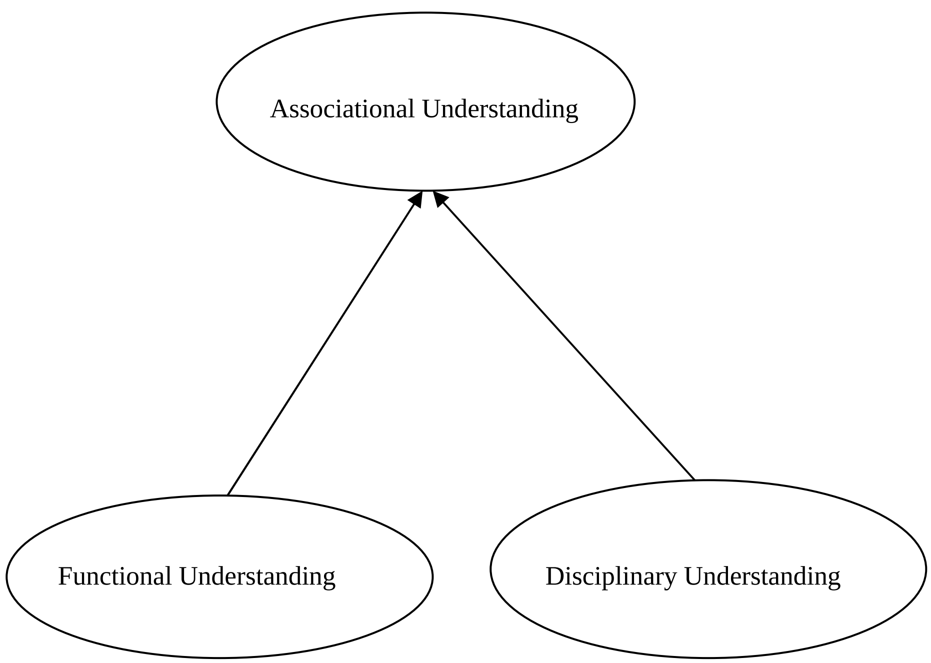
Relationship between associational, functional and disciplinary understanding. With regard to the relationship between the above first three types of understanding (conceptual, functional and procedural) and associational understanding, participants who achieve conceptual or functional understanding may be able to achieve associational understanding. This is because the pre-requisites needed in achieving associational understanding are the mastery of functional understanding and related engineering concepts or theories (disciplinary understanding. Thus, the absence of the mastery of disciplinary understanding means that the students may not be able to achieve associational understanding even if they are conceptually and functionally adept in the formulae learnt. However, participants with only procedural understanding will not be able to achieve associational understanding due to their lack of functional understanding in engineering mathematics.

> Sometimes, I cannot link the formulae to engineering questions [associational and disciplinary understanding] even though I can use the formulae to solve mathematical questions [procedural understanding]. (E09)

> Although I can do and understand the proof [formulae] all of the time, there are many times I cannot apply it in engineering situations, like my problem based assignments. (E02)

The diagram below shows the relationship among associational, functional and disciplinary understanding.

Figure 3. Relationship among associational, functional and disciplinary understanding



Relationship between solving engineering problem and associational understanding. Berlinghoff and Gouvea (2004), De Lange (1996), Ernest (2004), Peterson (1996), Putnam (1986), Stanic (1989), and Steen (1988) have underlined the fact that engineering and mathematics education have become indispensable in the social needs and economic survival of any modern society. Engineers solve real life problems in the society and mathematics is crucial in mastering it. Therefore, it is also essential that that the relationship between the five types of understanding and engineering problem solving is uncovered. From the data, it is uncovered that the key to the success of solving engineering problem solving is the achievement of associational understanding of mathematics learning. The comments below supported this conclusion.

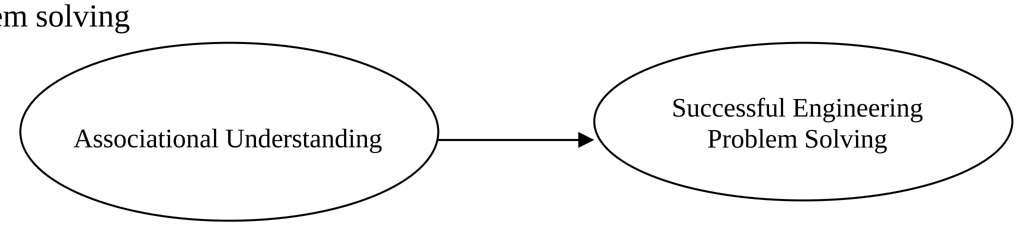
> Our engineering mathematics syllabus aims to impart mathematical skills that the engineering students can use in solving problems in engineering scenarios. They can know the steps or proofs of any formulae taught but if they cannot apply them in engineering contexts, these formulae are useless to engineers. (T02)

> Our other engineering modules' lecturers always expect us to be able to use what we have learnt in engineering mathematics in their modules because they said that it is this skill that will help us to solve the problems in their modules. (E20)

> A lot of times, when if we want to solve engineering related problem effectively, we need to know how to apply the math into the context of the problem. (E04)

The diagram below shows the important relationship between associational understanding of engineering mathematics and successful engineering problem solving.

Figure 4. Relationship between associational understanding and successful engineering problem solving



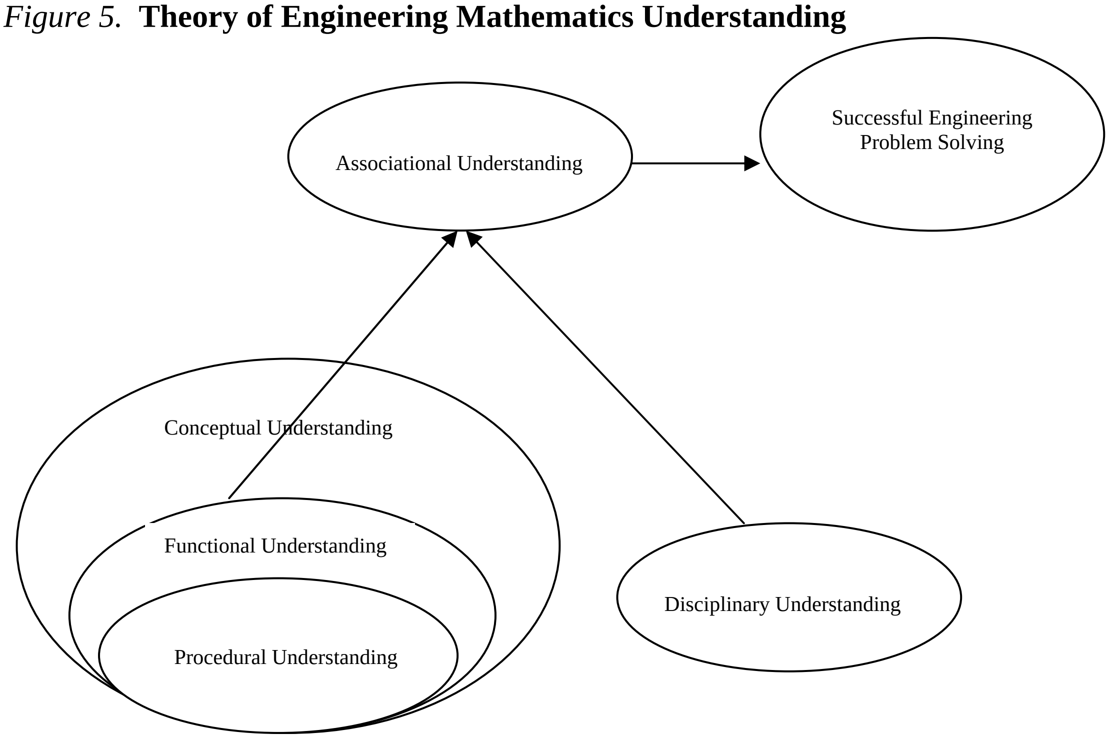
Theory of Engineering Mathematics Understanding

Through the relationships among the various types of understanding and successful engineering problem solving, the substantive theory of engineering mathematics understanding emerges. In the theory of engineering mathematics understanding, students understand engineering mathematics conceptually, functionally, procedurally, disciplinary and associationally. At the same time, these different types of understanding are closely related to one another in the students' understanding process in influencing the success of engineering problem solving to demonstrate the relationship between these five types of understanding, the example below is used as an illustration:

> Differentiation is an important mathematical concept and is represented by the formula: $\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. The participants who achieve **conceptual understanding** are able to understand the explicit assumptions and proof behind this formula. They also know that this formula measures the rate of change of y with respect to x and they will be usually utilised when rate of change is concerned in any mathematical problem (**functional understanding**). More importantly, they are able to compute a mathematical problem using the procedures involved in this formula such as $\frac{d}{dx}\left[x^n\right] = nx^{n-1}$ if the value of n is given (**procedural understanding**). The differentiation formula can be applied in engineering scenario where the relationship between current and capacitance in an electric circuit (**disciplinary understanding)** is represented by formula, i = C$\frac{dv}{dt}$ (i, C and v represent current, capacitance and voltage respectively). If they know this formula, they have achieved **associational understanding** as $\frac{dv}{dt}$ is an engineering concept that is converted from a differentiation function.

The diagrammatic representation below shows the theory of engineering mathematics understanding in this study.

Figure 5. **Theory of Engineering Mathematics Understanding**



Discussion

The literature review has shown that most studies that investigated on mathematical understanding have focused on Skemp's (1978) conceptual/relational and procedural/instrumental understanding. The theory of engineering mathematics understanding formed in this study also includes the categories of conceptual and procedural understanding as other studies claimed. Other than conceptual and procedural understandings, the theory of engineering mathematics understanding include three other forms of understanding that are uniquely present in engineering mathematics learning. They are the categories of functional, associational and disciplinary understanding in engineering mathematics learning. The presence of these three unique forms of understanding in engineering mathematics learning might be due to the fact that engineering mathematics focuses specifically on the learning of applied mathematical skills in the context of solving engineering problems. Engineers need to understand the functions of mathematical formulae (functional understanding) so that they are able to apply them in solving engineering problems (associational understanding) in relation with their disciplinary understanding of engineering concepts.

In this study, functional understanding refers to the ability of the students to understand the functions of the formulae in general mathematical terms. To some extent, functional understanding can be subsumed under Skemp's (1978) conceptual understanding that also encompasses the process of understanding the uses of the concepts. In this study, both conceptual and functional understanding are detached into separate entities, as it is revealed that only functional understanding is needed in the important associational understanding process. In order to utilise mathematics in engineering scenarios, students have to achieve disciplinary understandingthat give meanings to the functions of the mathematical concepts they have learned. This ability to convert mathematical formulae into engineering forms to solve engineering problems is termed as associational understanding in this study. It is different from that of Skemp as it is relating mathematical concepts and formulae to solving engineering problems while the one proposed by Skemp refers to the relationships between mathematical concepts.

The findings of the study reveal that the students who participated in this study generally focused on achieving procedural and associational understanding (that in turn, needs functional and disciplinary understandings) of engineering mathematics. Thus, conceptual understanding in engineering mathematics learning seems to be ignored. The possible omission of the learning of conceptual knowledge of engineering mathematics may well be related to the nature of mathematics as discussed by Schneider and Stern (2005). In general mathematics, learning, the abstract nature of mathematics requires the learners to constantly reflect on, and make inferences from, a mathematical formula if they are to understand it conceptually (Schneider & Stern). Thus, conceptual understanding may not be easily taught or learnt and it is exacerbated by the higher level of difficulty in engineering mathematics concepts (as compared to pre-tertiary mathematics) learnt in the case Polytechnic. In contrast, procedural understanding of mathematics demands rather less attention and cognitive input from the learners (Schneider & Stern). Thus, it may be perceived by these participants to be easier to teach and learn.

However, in the context of this research, the reasons why conceptual understanding is not pursued by most of the students in this study may be influenced by the wider social and educational context. First, the students recognized that attaining procedural, functional and associational understanding is sufficient for them to perform well in their tests and examinations. This is because the format of their examinations were based on the students' ability to understand associationally and procedurally. Thus, conceptual understanding is of no academic value to the students in examinations. In addition, they generally feel that conceptual understanding is irrelevant to them as they are not required to prove or justify the formulae they are expected to learn in their future engineering careers.

Nonetheless, there are some limitations to the research approach adopted in this study regarding the conceptualisation of understanding in engineering mathematics. First, although the research reported why the various types of understanding were pursued in the students' learning, it did not explain how these were formed. This was because the investigation of the category of understanding did not center on a micro, cognitive, process-based approach, which would have focused on how students perceived the actual ordered step-by-step process of mathematics problem solving. Thus, the research results could not clearly reveal the forms of understanding the students might potentially have achieved in the different engineering mathematics topics. It also could not explain why conceptual understanding in particular, and functional and/or associational understandings to a lesser extent, were not attained by some of the students. Neither did this study sufficiently clarify how the social contexts of the school (such as the lecturers' beliefs, pedagogies, and affective support, and peers' academic and affective support) as influenced by the curriculum, pedagogical or time constraints, might influence the development of the various kinds of understanding. Lastly, it is uncertain whether the five subcategories of understanding would be found in other domains of learning such as arts, humanities, business or languages. The confines of this study meant that the above issues fell outside its boundaries. However, these issues might well serve as possible topics to research in future.

Conclusion

The purpose of this grounded theory study is to uncover a central theory explaining engineering conceptions of understanding in mathematics learning, as perceived by the students and lecturers. In this regard, this study aims to answer the research question: "What are engineering students' conceptions of understanding in mathematics learning?" The research question was answered by the formation of the substantive theory of engineering mathematics understanding (comprising of conceptual, functional, procedural, disciplinary and associational understanding) in this study.

This study has deviated from the conventional forms of understanding of mathematics (conceptual and procedural) proposed by earlier studies. It has proposed the presence of three more types of understanding (functional, disciplinary and associational), especially in the context of engineering mathematics learning at diploma level. It has also shown how each form of understanding in engineering mathematics learning is related to one another. The data also shows that associational understanding is of utmost importance in solving engineering problems. To order to enable engineering students achieve a high standard of associational understanding in engineering mathematics learning, I would propose that the curricula should engage engineering problems as the basis where the concepts and formulae are taught. This will allow these prospective engineers to engage purposeful engineering tasks as part of their learning. This will thus enable them to build up their associational understanding of engineering mathematics. The affordances of tools such as technology can also be incorporated effectively in the engineering mathematics curricula to improve engineering students' associational understanding through effective visualisation and modelling of mathematical concepts.

I also do understand that changes in educational institutions are constrained policy issues in the wider political and social contexts. The pedagogical, affective and social influences from teachers and the types of engineering students in mathematics learning are also complex issues that need to be considered. Nonetheless, regardless of whether the proposals above are eventually adopted, in the spirit of social justice of education, at least, the awareness of the theory of engineering mathematics understanding can be created among the relevant educators.

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