MATHEMATICAL OPTIMIZATION

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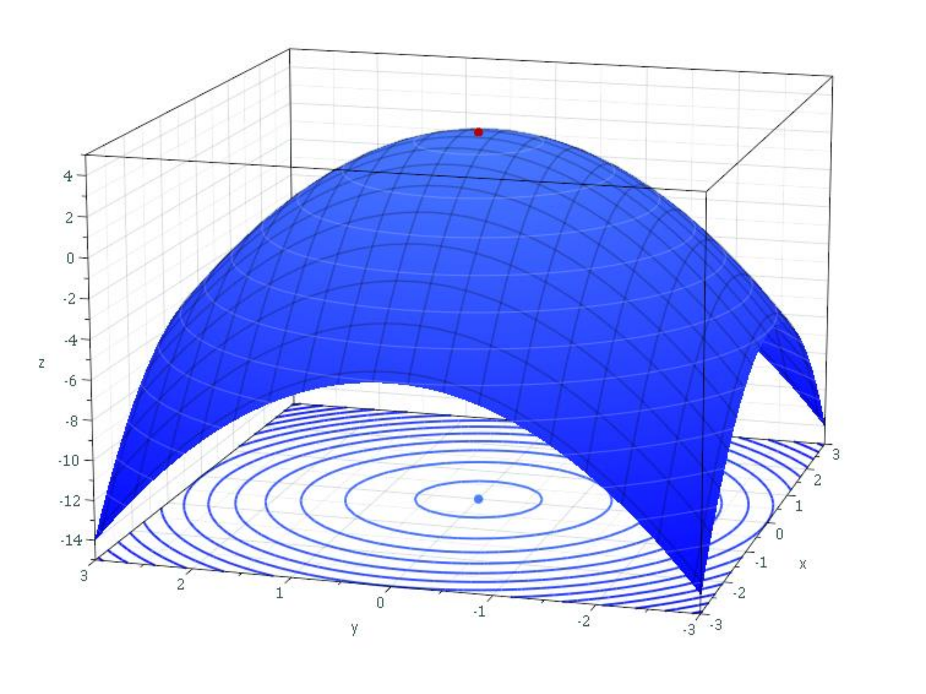
Mathematics Colloquium Series

Background Information

- Mathematical optimization - the selection of the best component from a set of available options
- Optimization problems consist of finding the maximum or minimum of a real function, known as an optimal solution
- Often involves a constraint
- Most widely used in the areas of mathematics, computer science, and operations research

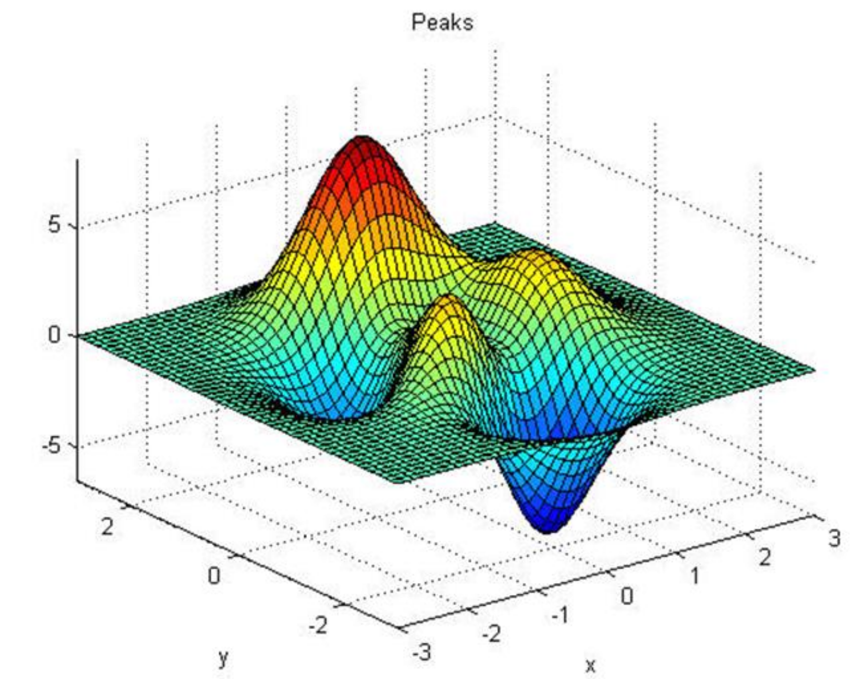


MATHEMATICAL APPLICATIONS

- Calculus of variations seeks to optimize an action integral over some space to an extremum by varying a function of the coordinates
- Global optimization - the development of deterministic algorithms that are capable of guaranteeing convergence in finite time to the actual optimal solution of a nonconvex problem
- Mathematical approaches to solving optimization problems include classical, linear & nonlinear programming, and game theory

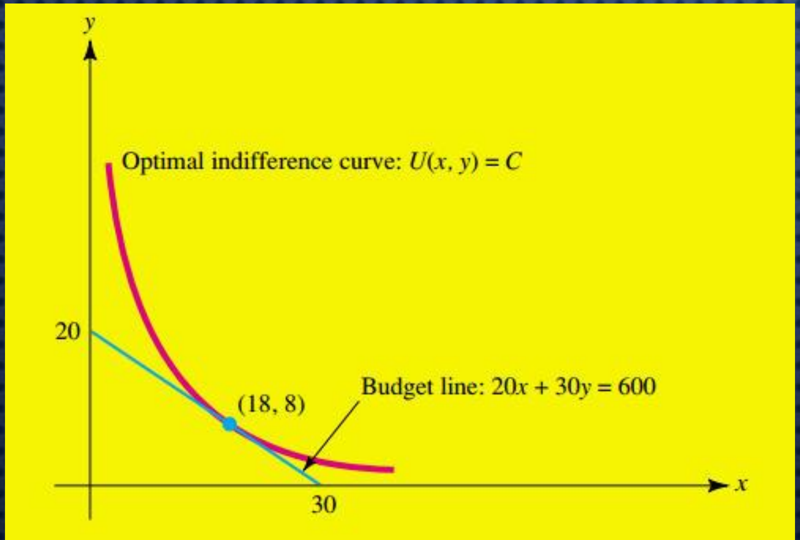
BUSINESS APPLICATIONS

- OPTIMAL ALLOCATION OF RESOURCES LIES AT THE HEART OF THE SCIENCE OF ECONOMICS
- CONSUMERS ARE ASSUMED TO MAXIMIZE THEIR UTILITY, WHILE FIRMS ARE USUALLY ASSUMED TO MAXIMIZE THEIR PROFIT
- ASSET PRICES, TRADE THEORY, AND THE OPTIMIZATION OF MARKET PORTFOLIOS ARE ALSO MODELED USING OPTIMIZATION THEORY
- MACROECONOMISTS BUILD DYNAMIC STOCHASTIC GENERAL EQUILIBRIUM (DSGE) MODELS THAT DESCRIBE THE DYNAMICS OF THE WHOLE ECONOMY

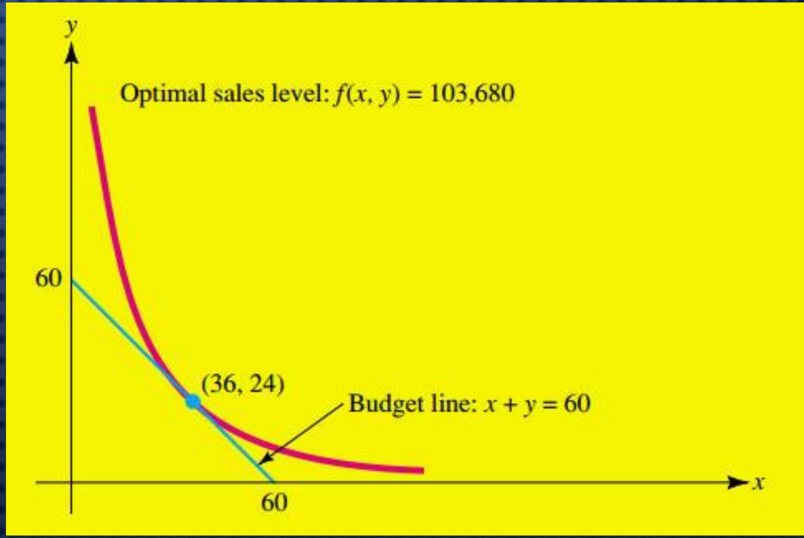


VISUAL EXAMPLES

The level curves of utility functions (indifference curves)



The relationship between the optimal indifference curve U(x, y) = C, where C = U(18, 8) and the budgetary constraint is 20x + 30y = 600

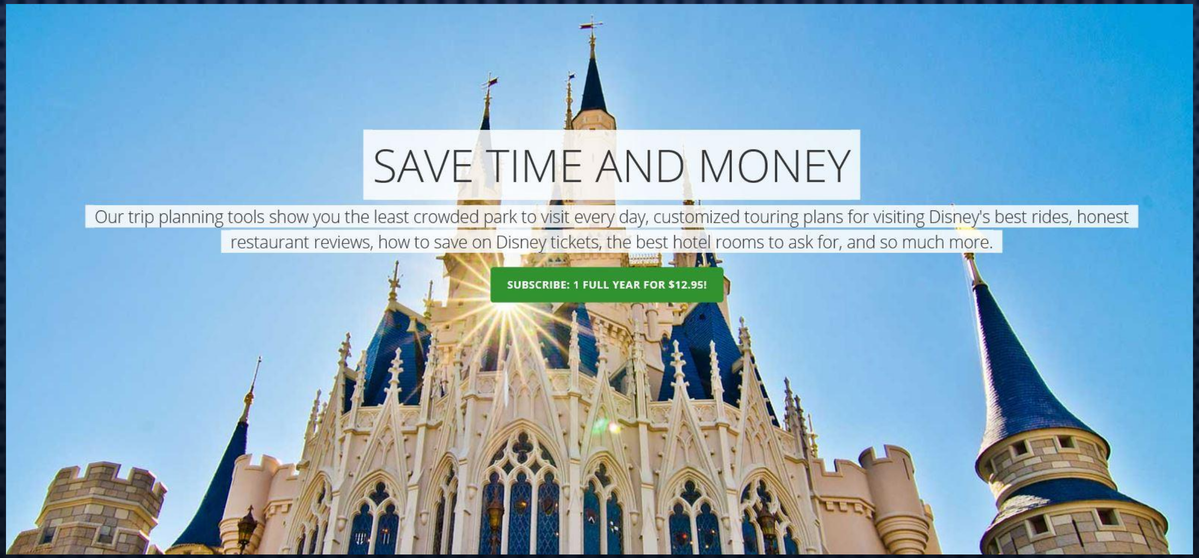


The relationship between the budgetary constraint and the level curve for optimal sales

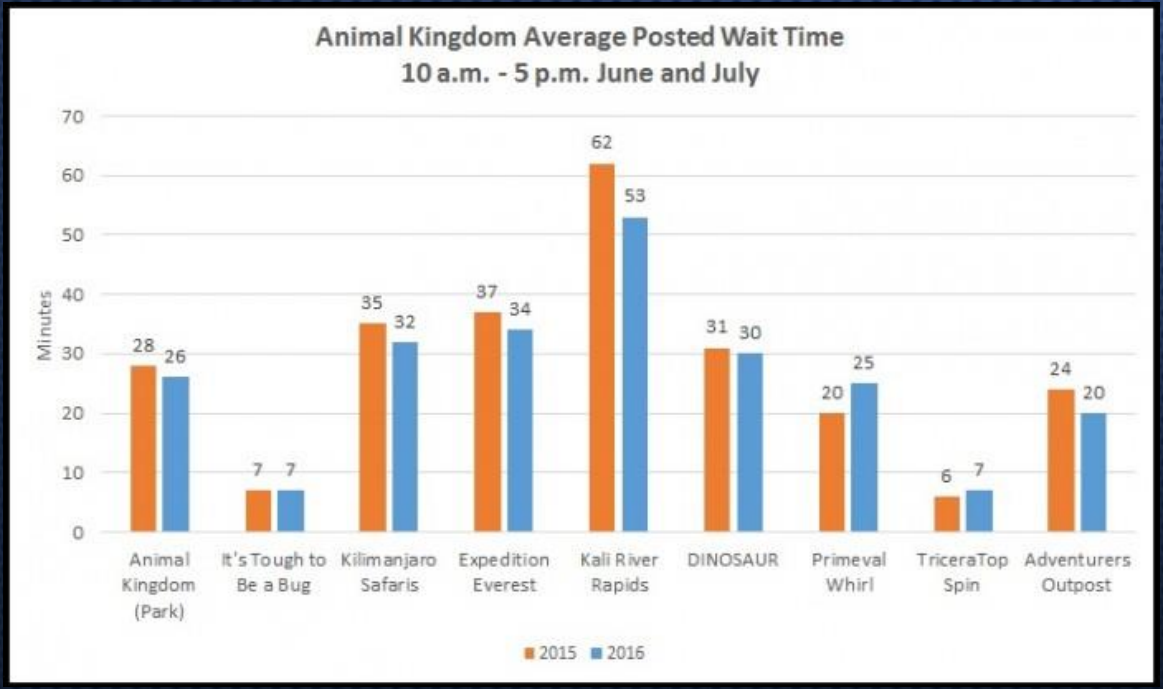
REAL WORLD EXAMPLE: DISNEY WORLD

TouringPlans.com

- Crowd Calendar shows how busy each Disney theme park is
- attendance across different weeks, months and seasons
- Customers can plan a park visit and avoid crowds with the help of:
  - Each park's opening and closing times
  - The park's Extra Magic Hours schedule
  - Any special events that might affect your visit
- shows yesterday's results - their predictions versus what actually happened



SUMMER 2016 AT DISNEY

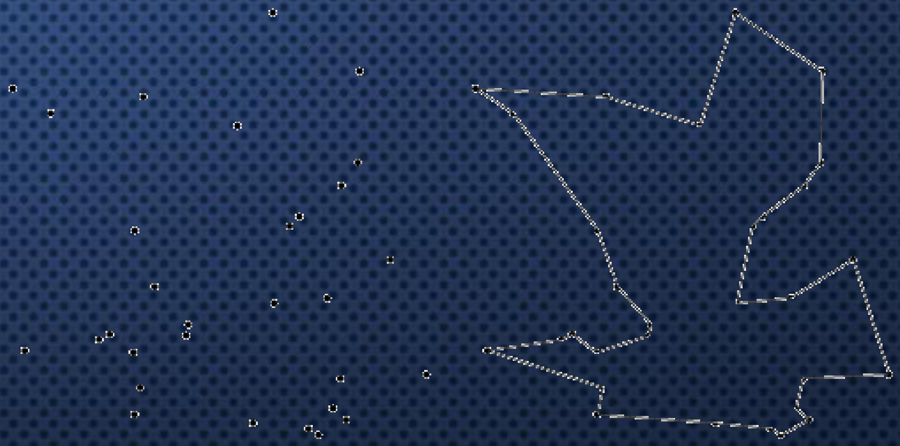


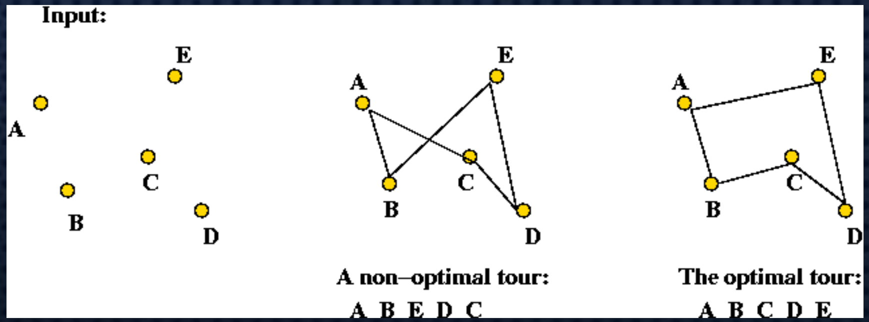
- Graph data based on standby waits, posted waits, and people in line between 10 a.m. and 5 p.m. (the peak time for crowds)
- Most major attractions at the Animal Kingdom has had a wait time drop in 2016
- attendance is lower for Epcot, Disney's Hollywood Studios, and the Animal Kingdom, but higher at the Magic Kingdom, versus the same period in 2015.
- Overall, attendance is slightly lower throughout Walt Disney World

MAIN ALGORITHM

- Traveling Salesman Problem (TSP)
  - Optimal route
  - better solution = cheaper solution
- Time Dependent Traveling Salesman Problem (TDTSP)

  the cost to travel from one city to another depends on:
  - The distance between cities
  - time of day of the travel





Algorithm: Avoiding Long Lines

- Q = computer time to come up with a result
- R = the set of all rides you want to ride
  - r = specific ride in R
- Edge E_{IJT} = walk from ride i to j at time t
- Start at the entrance and run a time-dependent Nearest Neighbor algorithm for each ride in R
  - each r_I in R is the ride visited after entering the park
- Save the set of all edges found in the paths into S
- Create a small number – random TSP paths for your rides – just put your rides in R in any random order to start with. For each path in P, calculate the "cost" of the path

Algorithm Continued

- While (we still have time according to q)
- Pick 2 paths (parents) from P using tournament selection
- Pick a genetic operator such as:
  - Random Mutation
  - Time-Dependent Random Mutation
  - Lin-Kernighan
  - 2-opt
  - Cycle Crossover
  - Brute Force Permutation
  - Fast pass Mutation
- Apply the chosen operator to the parents. The path that is created by this operator and the parents is called the child
- Calculate the cost of the child
- If the child's cost is less than the cost of the worst path in P:
  - Delete the worst path in P
  - Add child to P
- If we've gone a really long time without adding a child to P:
  - Delete all but the 1 best path in P
  - Create new, random paths for all of the remaining space in P
- done // while (we still have time..)
- Send the results back to the server

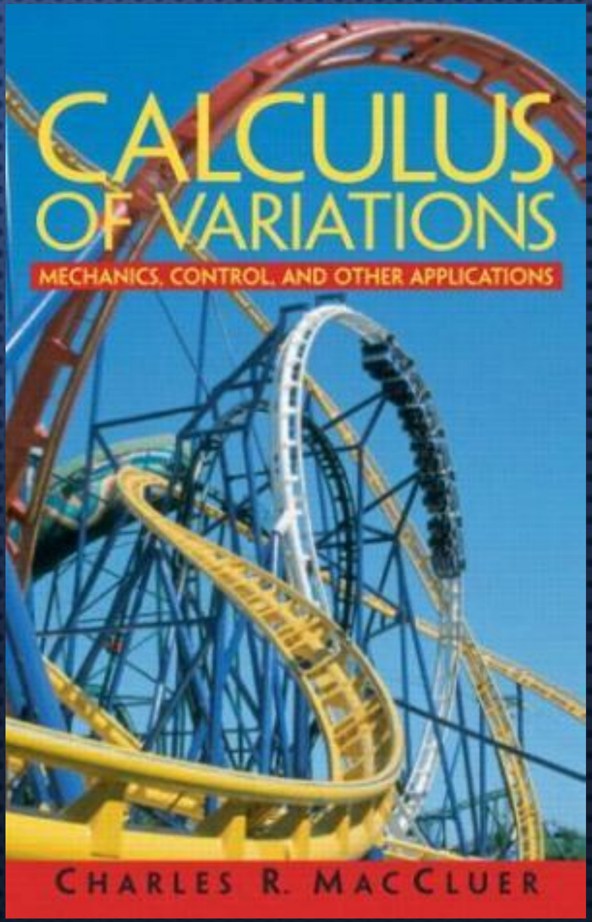
HOW MANY POSSIBILITIES ARE THERE?

Game	Ways to Play	Like a Touring Plan with
Tic-Tac-Toe	31,896	8 attractions
Connect 4	4.5 x 10^12	15-16 attractions
Checkers	5 x 10^20	21-22 attractions
Chess	10^40 to 10^50	35-42 attractions
Go	10^170	106 attractions

- Magic Kingdom has 43 attractions, ranking between the game of Chess and Go in terms of complexity
- Takes into considerations food places and shows
- Variations for different times of the day or year
- Takes a lot of computing power
- There are 10^{170} possible moves in the game Go, while only 10^{80} number of atoms in the observable universe

Calculus of Variations

- The feasible points that satisfy the constraint form a polygon
- The edges of a polygon – edges of the park area
- Each ride is similar to a vertex on a polygon
- The extrema occur at the vertices
- Many similar problems involve linear programming



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