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Blazing New Pathways for Digital Engagement in Mathematics Using GeoGebra, Math Manipulatives, and Picture Books in a STEM World to Nurture Confident Young People in Mathematics

Cover Page Footnote

This paper is part of a conference proceeding and presentation for the Florida Distance Learning Association and Florida Association of Mathematics Teacher Educators Annual Conference, February 9, 2024.

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Abstract

Using best practices for teaching mathematics like GeoGebra, math manipulatives, and children's literature picture books can all be connected to each other and better help to prepare students for a STEM world to develop confident young people in doing mathematics in a world of AI and emerging technologies. This paper will highlight all the above best practiced and share how they can be connected and taught in a way that is make math meaningful and help students to be confident with technology and mathematics in a way that helps them see math in a hands-on, visual, and abstract way. Learners can be digitally engaged with GeoGebra, Virtual and/or actual math manipulatives, and children's math literature/picture books. This paper will give an overview of the theory of these best practices, discuss manipulatives, the use of GeoGebra software in learning mathematics, and show the connections to the picture books and everyday life math skills. The goal in using such best practices in mathematic is to develop confident young learners for a STEM world where so much of the technology now focuses on AI and emerging technologies to motivate learners, creating blazing new pathways for digital engagement in mathematics learning.

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Keywords

Math Anxiety, GeoGebra, Manipulatives, Children's Math Literature Picture Books, STEM

Introduction

Math is stimulating, challenging, enjoyable and simple, effective and expansive and the more we can use hands-on tools to show this to kids, the more we can share this love of the mathematical universe with them. - Esther White

Dealing with anxiety of mathematics is an actual phenomenon and many young learners are confronted with this when learning math at various grade levels Kindergarten through College today. Many people often go through such math anxiety through their whole life and it can often affect decisions in life as well as the career selections one will make. It is imperative that all people feel confident in their ability to do math in an era that depend on so profoundly on STEM areas and problem solving. It really is a school's obligation to see that their students' value and feel confident in their ability to do math and use

technology to learn because ultimately, all choices individuals make and choosing of careers may be resolute in part by their dispositions toward mathematics. Math anxiety is a real phenomenon that has been researched for many years now (Alday and Panaligan,2013; Beilock and Willingham, 2014; Boaler,2008; Quander, 2013; Richardson and Suinn, 1972; Scieszka and Smith, 1995; Williams, 1988). Mathematics educators need to take it seriously and use research to address the problem in an age of STEM. Resources and websites for addressing math anxiety and improving attitudes as well as incorporating technology like GeoGebra, math manipulatives, photography, and children's literature books will all be shared and included in the paper. Today it is critical we help to create mathematically confident young people in the STEM/AI world we now live in.

Anxiety with Math is a Real Phenomenon and Many People Suffer from It

Apprehension toward math continues to devastate our society and distresses our young peoples' success and achievement within this subject area of mathematics (Finlayson, 2014; Quander, 2013). Quander (2013) found elementary teachers need to help make students to be lifetime learners and advance a productive mathematical temperament so that they are prepared for advanced schooling and eventual careers, many which may be STEM related. Math anxiety can impede not only mathematical performance but also interest and then career choice and many decisions in life. The awareness as educators to exam math anxiety levels, enthusiasm to learn mathematics, and using advanced technologies like GeoGebra to instruct and motivate learners is critical today in a global society of STEM and also can impact achievement areas of learners (Furner, 2019, 2022; Gonzalez-DeHass et al., 2017 and 2023; Furner & Marinas, 2016; Furner & Marinas, 2020).

Many educational organizations like the National Council of Teachers of Mathematics (NCTM, 1989) believe that mathematics teachers need to assess students' mathematical disposition regularly regarding: checking for confidence in using math to solve math problems, communicate thoughts, and reason; being adaptable in exploring mathematical concepts and employing a variety of approaches when solving problems; preparedness to persist in mathematical problems; interests, inquisitiveness, and ingenuity in doing math; student ability to reflect and monitor their own thinking and performance while doing math; and value and appreciate math for its real-life application, connections to other subjects and norms and as an instrument and idiom of its own.

Young learners often say: "I like the math class because of the teacher" since the math teacher knows how to present developmentally the subject matter, creates a learning atmosphere advantageous to learning with empathy, has high expectations for all students without regard to gender, race, or language barriers, and uses a variety of assessment methods and teaching styles to better reach all students to address math anxiety and better ways to teach and reach students (Schoenfeld, 2022; Chernoff & Stone, 2014; Dowker et al., 2016).

Today it is critical to know that there are two distinctions to math anxiety handlings: prevention and reduction and there are distinct strategies and methods to address each is different ways. Furner (1996) synthesized three steps to prevent math anxiety: 1). Engaging in using "Best Practice" in teaching math like using: math manipulatives, cooperative learning, dialogue of math, enquiring and conjecturing, justifying one's thinking, math journaling, using a problem-solving approach to teaching, interdisciplinary instruction of content, emerging technology, assessment as an integral part of instruction, etc.; 2). Incorporating the NCTM and State/Common Core Math Standards into the curriculum and/or instruction; and lastly, the importance of discussing feelings, attitudes, and appreciation of mathematics with students. Research by Furner (1996) also found that there are three approaches to reduce/lower math anxiety: 1). Psychological procedures such as math anxiety managing, desensitization, therapy, group support, bibliotherapy, and deliberations/conversations about experiences; 2). As a math anxious learner feels less fear and dread toward math, he/she might begin to build their self-confidence by taking more math classes and exposure to higher level math concepts; and 3). Most research on math anxiety reduction has shown that until a person with math anxiety has confronted this math anxiety by some form of discussion/counseling no "best practices" for teaching mathematics will help to overcome this fear of the subject.

Math teachers during the school year while teaching mathematics should use some advantageous instructional methods which are advocated now for teaching mathematics using the Concrete-Representational-Abstract (CRA) Model for teaching mathematics as follows: First educators need to tart with the Concrete using hands-on manipulatives like Geoboards, then secondly, they must move to Representational models in diagrams (or use Virtual Manipulatives like NLVM at: http://nlvm.usu.edu/), and lastly, connect to the Abstract symbolism where student understand and function at an abstract level completely (GeoGebra software works well at: http://www.geogebra.org/cms/en/). The CRA Model is really the bases for the best practices pedagogy for teaching mathematics starting with young people, but should also be used at all levels of math instruction. See Figure 1.

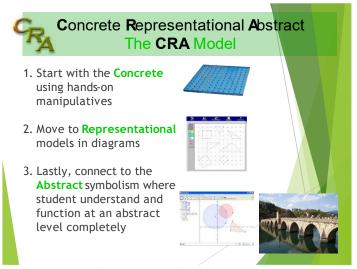


Figure 1. The CRA Model for Teaching Mathematics

The book author Jones (2012) found as discussed in her book, *Visualizing Mathematics*, that it is essential that math teachers help students envision and create images of their mathematics comprehension so young people view math everywhere and as a large part of their being. Beilock & Willingham (2014) in their research have found that math teachers can help to address and reduce math anxiety. The author believes by using technology like GeoGebra along with the photography and children's books, teachers can make better connections and students are going to be more highly motivated to learn math (Furner & Marinas, 2014; Marinas et al., 2016).

Defining Math Anxiety in a STEM/AI World

Mathematics anxiety can be defined as a dread of math which interferes with working with numbers and solving math problems or daily life experiences that involve numbers or mathematics. The NCTM recognizes math anxiety as a problem and specifically included in its assessment practices. Standard #10 from the NCTM prompts teachers to assess their students' mathematical dispositions; for example: self-confidence in employing math to solve word problems, communicate concepts, and reason mathematically. Math anxiety is often caused by a amalgamation of internal and external influences; nevertheless, educators cannot modify internal factors within the learner, so as teachers it makes sense to concentrate on what teachers can control (Chernoff & Stone, 2014). Math anxiety has been researched for almost fifty years and is a universal phenomenon, unfortunately not enough is being done today in schools to address it in how we approach teaching mathematics (Beilock & Willingham, 2014; Dowker et. al, 2016; Geist, 2010). Poor dispositions toward math and carrying math anxiety around with you in life are grave impediments for students at all levels of schooling nowadays (Geist, 2010). Beilock and Willingham (2014) found in their study and summarized: "Because math anxiety is widespread and tied to poor math skills, we must understand what we can do to alleviate it" (p. 29). Poor teaching approaches are not the only cause of math anxiety. Because math anxiety can be seen in daily living activities as well as in class work or assignments, the need to have a multi-pronged approach is crucial to addressing it. Applying anxiety-reducing techniques in a multitude of activities and recurrently through instructional activities aids to address a variety of learner needs. This is like the idea of applying different management and organizations skills suited the situation. Research from Skagerlund et al. (2019) discovered that math anxiety can weaken math ability, they propose learners need to acquire approaches to bring about this so that it does not affect their working memory and number processing when they are doing math. Using different approaches before teaching a math activity allows the teacher to set a more focused and less anxious tone for a math learning using a variety of approaches to teaching. Employing procedures that lower anxiety and provide support just prior to beginning the math activity, as well as during the activity help signal to the anxious learner to a more positive approach to math school work and the subject.

Stopping Math Anxiety in Students

Today there are several approaches schools can follow to help thwart math apprehension. In cooperation educators and parents play an important part in aiding to nurture positive attitudes toward mathematics. In many intervention programs, early intervention, and action aid to foster positive mathematics dispositions. Today the math education field has made more of a push to increase and reassure math literacy in schools, and laterally with that drive has established valuable resources to boost math capability. Research by Mammarella et al. (2018) show how important it is as instructors to detach the math from the anxiety levels and in their investigational results showed that children with severe math anxiety, nonetheless through no changing dyscalculia were explicitly impaired in the hands-on interference task, while learners with developing dyscalculia (with or deprived of mathematics anxiety) botched in the working remembrance duties. The above research findings contend how critical it is to distinguishing between the cognitive processes underlying the profiles of a child, which can have factors as teachers report preventative and lessening strategies as it correlates to math anxiety levels. A successful program established by the Southeastern Consortium for Minorities in Engineering (SECME) is in schools today for high minority populations to stimulate and grow students interested in math, science, and engineering fields. SECME was initially an acronym for Southeastern Consortium for Minorities in Engineering. The organization is based out of Atlanta, Georgia at the Georgia Institute of Technology. SECME is a deliberate coalition to recommence and fortify the professional expertise of K-12 educators, to motivate and counsel students, and invest parents so that all their children can learn and achieve at higher echelons. (SECME, n.d.) Many teachers find this program very useful to turn young people on to math and motivate them to like the subject more. The grades K-8 school years are critical to instilling confidence and powerful attitudes toward math in young people. Deterrence of math anxiety in students is all about instructor planning and employing the best possible teaching strategies in mathematics instruction (dos Santos Carmo et al. (2019). The way math anxiety is fixed in our schools, too put it simply, is better teaching to reach all students. Research by Finlayson contend using a constructivist style of instruction which emphasizes the following practices:

- Using whole group instruction first then
- Quest for student queries and interests
- Key resources should be manipulative materials
- Learning should be interactive constructing and building on what learners already know
- Instructor interacts/negotiates with students
- Evaluating students through observations, interview, tests, etc.. The process is as critical as end result
- Knowledge is dynamic/change with experiences
- Students work in groups" (Finlayson, 2014)

Lessening/Overcoming Math Anxiety and Building Confidence is Key

Math anxiety reduction is much different from the prevention of such anxiety. While every educator would like to prevent a student from experiencing math anxiety, some come to school afraid and worried about learning math. Many math educators contends that a

person who suffers from math anxiety needs to first lay the groundwork by coming to terms with their feelings and challenge their present views and comprehend they are not unaccompanied; secondly, a person has to change their views and negative thought and use intervention approaches to progress one's discerning that they may be efficacious with math; thirdly, a person needs to know themself, it is critical that a person knows his/her learning style/approach and that he/she apply such methods to doing math by successful people; and lastly, when a student has increased confidence and approaches for undertaking math then they must apply what they learned and how they actually go about doing the mathematics. Additionally, the problem for those who suffer from math anxiety is the condition of anxiety itself. Research by Rubinstein et al. (2015), found apprehensive learners often focus on negative impetuses more than positive stimuli, essentially making themselves more uneasy. The equivalent is true of individuals with math anxiety; the one difference is that for individuals with math anxiety, math is the negative stimuli (Rubinstein et al., 2015). From this it is suggested that math anxiety may be remedied through treatments designed to lower anxiety, such as cognitive social therapies and exposure therapy (by exposing a someone incrementally to that which they are fearful like doing math) (Rubinstein et al., 2015).

Educators need to realize that there are important supportive techniques in a counseling setting when working with the math anxious. For example, some researchers (Ramirez et al., 2018) propose systematic desensitization as an effective approach for helping people reduce their math anxiety. Systematic desensitization in the framework of math anxiety which can be a distinct and measured gradual exposure to math ideas that are producing students to develop anxiety and teaching learners how to manage such distress. When using systematic desensitization, a mutual practice in counseling today, learners come to understand that their math anxiety is a learned conditioned behavior, one they were not born with, and they can be trained to overcome it by constantly applying their self-monitoring strategies to become less math anxious. Some researchers advocate the use of relaxation in conjunction with repeated positive messages and visualizations to reduce math anxiety.



Figure 2. Math Manipulative for Teaching Mathematics

Operating from the academic perspective, Zemelman et al. (2012) summarizes much evidence-based practices for teaching math which include: (a) use of manipulatives (make learning math concrete) [See Figure 2]; (b) use cooperative group work; (c) use discussion when teaching math; (d) make questioning and making conjectures a part of math; (e) use justification of thinking; (f) use writing in math for: thinking, feelings, and prob. Solving; (g) use problem-solving approach to instruction; make content integration a part of instruction; (h) use of calculators, computers, and all technology; (i) being a facilitator of learning; and (j) assess learning as a part of instruction.



Figure 3. Children's Picture Books for Teaching Mathematics

Connecting Manipulatives to Picture Books and GeoGebra

"I think that children's literature offers a wonderful vehicle for helping teachers teach math well."

-Marilyn Burns

"If you want your children to be intelligent, read them fairy tales. If you want them to be more intelligent, read them more fairy tales."

-Albert Einstein

As stated above it is important that math teachers read the children's book and fairy tales like *Sir Cumference and the Dragon of Pi, a Math Adventure* by Neuschwander-See Figures 3-4), the son whose name is Radius (his title is referred to but not labelled in math terms) protects his father from dying, Sir Cumference, after unintentionally turning him into a fire breathing dragon. A math formula/equation is the solution to the problem here in this book. In the story Sir Cumference names Pi in this story. The story in the book tells how the math formula for \prod was uncovered? The book offers a fictional story nevertheless the author's purpose is achieved when young people reading this book recall this math lesson and what Pi actually means.

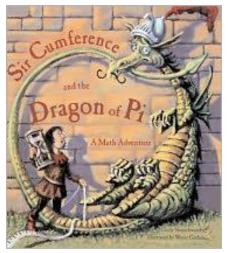


Figure 4. Sir Cumference and the Dragon of Pi Book

Assisting Young People to Gain Math Confidence for a STEM World

We are living in an age of advancing technologies that are constantly changing. Children need to be literate as well as be very good at mathematics and problem solving in order to compete in a global society. A youngster's lack of confidence and ability to do mathematics may impact his/her entire life continually both in all choices they decide on a regular basis as well as forthcoming vocational choices. Educators in an age of STEM should be prepared to reach all learners and develop their confidence and ability to do mathematics so they can compete globally. Teachers today should check to see that all their students have positive attitudes and dispositions toward math (NCTM, 1989). It is very important to ensure our young learners are confident and well equipped in math in a STEM world if they are going to vie for such high-tech jobs now and in the future. Today, the United States

and other countries are working to lead more young people into the fields of Science, Technology, Engineering, and Mathematics (STEM) so as countries all can better compete globally. Today it is critical teachers build math confidence in our students, educators need to address directly the matter of math anxiety as it manifests the issue as uncertainty or learned helplessness in observed math achievement. Many adults do not like mathematics. Sparks (2011) contends that as the STEM fields become more significant for our young people to study, our schools and teachers need to do more to address math anxiety levels in learners so our young people are confident in their ability to study fields associated to STEM areas.

Children's picture book like, Math Curse, by Scieszka and Smith addresses the issue of math anxiety. The book is a great resource for educators to help anxious learners come to terms with the truth that not all individuals feel self-confident in their capability to do The book commences with the teacher Mrs. Fibonacci, who says to her mathematics. students that they can think of almost everything as a math problem, one student then starts thinking and worries and becomes overcome by the breath of math. His math anxiety then becomes a real curse, hence the title of the book, Math Curse. Nevertheless, the character in the book ultimately comprehends that math is all around us and there is no way of avoiding it in daily life; therefore, the math anxious youngster in the story recognizes math as a means of making one's life easier. *Math Curse* may be used as a form of bibliotherapy to prompt discussion on the topic of math anxiety and allow other students to discuss their feelings on the theme to comparisons to the character in the story. Isdell (2017) wrote another great book, A Gebra named Al, about a young girl who struggles with her feelings toward math at the middle school level. This is also a wonderful book to incorporate in a bibliotherapy lesson to address math anxiety with students. Hebert and Furner (1997) feel that teachers need to take the time in their math instruction to address such affective aspects of learning mathematics so that students can come to terms with their feelings toward mathematics. Sripatmi et. al (2023) conclude that using picture/children's books and such media help reach students better in the teaching of mathematics.

State Math Standards as they Relate to using GeoGebra

Most schools and states in the USA today are adhering to the new Common Core Math Standards (Council of Chief State School Officers and the National Governors Association Center for Best **Practices** (NGA Center) which can be found http://www.corestandards.org/ When math teachers relate real-world problems through the use of dynamic technology like GeoGebra and connecting them to photography to make significant correlations in mathematics, students can recognize that geometry/shapes and mathematics surround us. GeoGebra is an ideal piece of math software to use to teach many of the Common Core or State Standards today. Furner and Marinas (2016) offer many premade GeoGebra activities that match up with the Common Core Math Standards and with many more resources at www.matharoundus.com and there are also many GeoGebra resources for teachers at: geogebra.org. GeoGebra is great software that can even involve programming and as a technology on the computer like gaming can be a powerful aid to motivate learners.

Technological tools being used as part of instruction is critical in today's world of STEM. Young people need to learn to succeed in math at much higher levels of generality, represent and solve multifaceted problems, and emphasize decision-making and reasoning more (National Council of Teachers of Mathematics. NCTM believes that mathematical power can arise from technology, which includes increased opportunity for learning, increased opportunities for real-life social contexts, and orientation to the future, this also connects with gamification aspects by employing technologies to motivate learners. As part of the President's Council of Advisors on Science and Technology (PCAST) by Holdren et al. (2010) they issued a policymaking report with explicit commendations to government leaders given to safeguard that the United States is a frontrunner in Science, Technology, Engineering, and Mathematics (STEM) education in the upcoming decades. One major recommendation is to recruit and train upwards of 100,000 new STEM middle and high school mathematics teachers over the next decade that are able to prepare and inspire students to have strong majors in STEM fields and strong STEM content-specific pedagogical preparation for such fields. PCAST believes that teachers are the most important factor to address in ensuring excellence in STEM education of future young people for our nation. Despite the ongoing efforts to promote the use of technology in education (e.g., National Council of Teachers of Mathematics [NCTM], National Educational Technology Standards for Teachers [NETS*T], educators' ineffective usage of technology skills has been cited in the research.

Software like GeoGebra, is a multi-platform dynamic mathematics software for all levels of education from elementary through university that joins dynamically geometry, algebra, tables, graphing, spreadsheets, statistics and calculus into a streamlined and integrated software package. This free dynamic mathematics software that can be downloaded free and accessed immediately at: http://www.geogebra.org. GeoGebra allows students and teachers the freedom to use it both within the classroom and while at home or on the go. GeoGebra has a large international user and developer community with users from 190+countries it has currently been translated into 55 different languages.

GeoGebra can be employed to demonstrate how mathematical formulas/equations can be used in commonplace ways modeling math. Research by Aydin & Monaghan (2011) uncovered that math teachers must explore the potential for learners to view mathematics in the real world through coding mathematical features of digital pictures using a dynamic geometry program like GeoGebra software. Mathematics teachers may find the following videos (Mathematics and Multimedia, n.d.) of basic training for GeoGebra at: http://mathandmultimedia.com/2011/01/01/geogebra-essentials-series/ useful as they provide great resources for how to quickly use GeoGebra in their math classrooms.

Math Manipulatives, Children's Picture Books, and GeoGebra Clarified Practice

Using a hands-on approach and math manipulatives (physical didactics or virtual) are common in most math lessons today in schools in the USA and around the world. Concrete manipulatives are critical for students to developing understanding of math concepts

(Furner & Worrell, 2017). Moore and Rimbey (2021) found that math manipulatives help better connect the math ideas for better understanding. Iqbal et al. (2021) found that using math manipulatives had a positive impact on student achievement in learning mathematics. Larson and Rumsey (2018) contend that children's literature in mathematics brings stories to life when teachers integrate literature and math manipulatives to make math lessons meaningful as shown in Figures 8, 9, and 12. Figure 8 shows concrete Tangrams, Tangrams on GeoGebra, and a children's book using Tangrams. Figure 9 provides Color Tile manipulatives to model a math problem in the book, *Spaghetti and Meatballs for All* written by Marilyn Burns, there is also a GeoGebra model of the problem shown. Furner (2018) found that using children's literature to teach mathematics was an effective vehicle in better reaching all students. Washington (2023) in her review of teaching mathematics creatively discusses the importance of using technology like GeoGebra, manipulatives and virtual forms, storytelling and children's literature in the teaching of mathematics as some of the best pedagogy for teaching the subject in a creative manner to better reach students.

Mathematics teachers may ask why it is important to make connections and motivate students about learning math while using GeoGebra? Teachers will find that when using GeoGebra, educators will be able to: demonstrate a purpose for mathematics; develop relations between math ideas and shapes and; this software will show practical applications to math in life; it employs innovative teaching in the classroom; it stimulates through photography/modeling; it employs emerging technologies in math with many real world application; GeoGebra can aid in addressing math anxiety as a motivating form of technology so that students feel confident for all STEM fields when they complete high school.

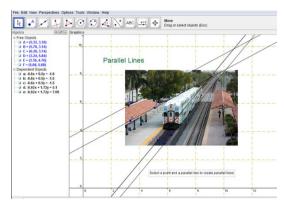


Figure 5. Parallel Lines in a Photo with GeoGebra

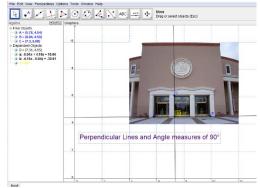


Figure 6. GeoGebra GGB demonstrates Perpendicular Lines on a Photo Perpendicular lines create right angles, 90-degree angles, and like in the GeoGebra file photo above [See Figure 6] right angles and perpendicular lines are drawn on the photo with the GeoGebra software all allowing students to identify vocabulary and math ideas (Furner and Yahya, 2020).

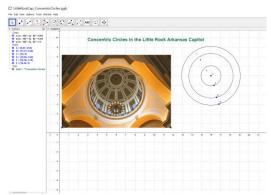


Figure 7. Circles and Concentric Circles Done in GeoGebra

The photo in Figure 7 was imported into GeoGebra and then students were asked to draw **circles** and **concentric circles** like seen in the photo of the Little Rock Capitol building.



Figure 8.Tangrams in GeoGebra, Children's Book, and Manipulative Lesson

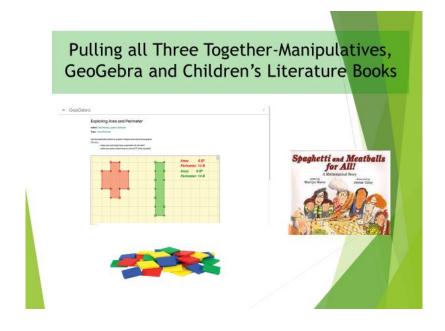


Figure 9. Color Tiles with a Book Problem Using GeoGebra

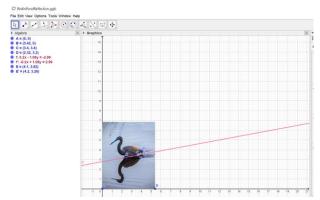


Figure 10. GeoGebra File Showing Photo and Line of Reflection

Reflections can often show up when taking photos of water, glass, or any other type of reflective surface. The photo above in Figure 10 shows a photo of a **reflection** of a bird in water with a **line of reflection** draw in GeoGebra.

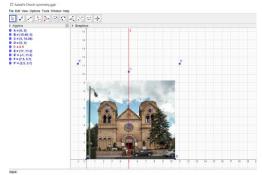


Figure 11.GeoGebra File of Symmetry of Photo of a Building with Symmetry Figure 11 shows a photo inserted into the GeoGebra software, a line was drawn through the center of the photo and then a **point** was selected and **reflected along the line** to show them as **symmetrical** to each other.

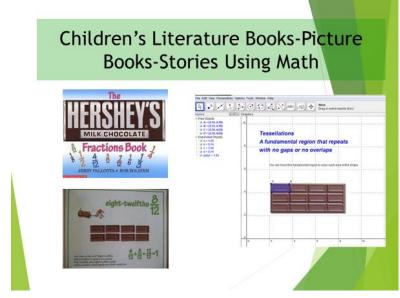


Figure 12. Photos of Tessellations-Using Chocolate Bar, Book, and GeoGebra

Tessellating patterns are patterns that repeat with the same **fundamental region** covering a space, with no **gaps** and no **overlaps** like seen in the chocolate bar rectangular pieces seen in Figure 12. Students can use the GeoGebra and move the pieces to test their hypothesis of the same shape repeating. By using GeoGebra and inserting photos into the software, students can then use the tools in GeoGebra to do the math, learn the vocabulary, and start better understanding mathematics concepts better.

Concluding Remarks

Mathematics educators need to look deeper at their students' needs and address the math anxious students they have in their classrooms today to better prepare them for our hightech world we now live in. As educators, we need to better prepare our young people for STEM, using technology, and having a strong and curious interest in mathematics. Math is best learned especially in the elementary levels when teachers use the CRA model starting with concrete and manipulatives first. They need to make the connections to the representational through computer and GeoGebra with representational models and then getting to the abstract. Young learners will enjoy using technology like GeoGebra today while learning mathematics in a very meaningful and motivating way. Reading children's literature math books to students helps them see value and understanding of many math concepts in the real world with real world application. Math manipulatives, children's literature, and technology like GeoGebra are the keys to success for students when learning mathematics and it also better prepares them for a STEM world where they are confident in their ability to do mathematics. Manipulatives can provide the concrete experiences for students, children's books offer great representational models of the math used in everyday life, and GeoGebra allows students an opportunity to use the math at abstract levels while employing the emerging technology. All of these practices mentioned in this paper are considered best practices for teaching mathematics today. Math educators can also do all with technology and through distance learning using the virtual math manipulatives, the YouTube picture books stories, and using GeoGebra online if they are teaching entirely online.

Educators today have the great responsibility to see that their students feel confident in their ability to do math and see a purpose for it in life, as ultimately a child's life: and all life decisions they will make and vocations may be determined based on their temperament toward mathematics. As math teachers we must make the difference in our children's' feelings toward math while preparing them for a future with a greater STEM emphasis. In the STEM world we live in now, it would be wonderful to see more young people when inquired on how they feel about mathematics say, "Math is my preferred course at school" or "I am great at geometry!" or "I can solve almost any word problem!" Math teachers today when employing all the "best pedagogical practices" and engaging math confidence building techniques in our schoolrooms today, educators and schools can produce more mathematically self-confident young people for the 21st Century STEM world we now live in. Math manipulatives, children's math literature, and technology like GeoGebra are a large part of learning today and used to help cover math content as well to be a motivating

factor in learning preparing our young people for a STEM world. These are some of the best pedagogical practices today for teaching mathematics to prepare young people for STEM.

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