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Creative Geometry Games

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by

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Introduction

The geometry game presented in this article was inspired by Bright and Harvey's (1988) *Learning and fun with geometry games*. In their article, Bright and Harvey (1988) propose three interactive games: Polyhedron Rummy, Polygon Rummy, and That's Stretching it. Polygon Rummy is a game pertaining to plane geometry where the instructional objective is to construct a figure using lines and angles, whereas the goal of polyhedron rummy is to construct a solid using faces and angles. Although Bright and Harvey focused on constructing shapes, they also suggested presenting pre-constructed shapes to students, allowing them to classify and identify them (Bright & Harvey, 1988). Our activity does just this.

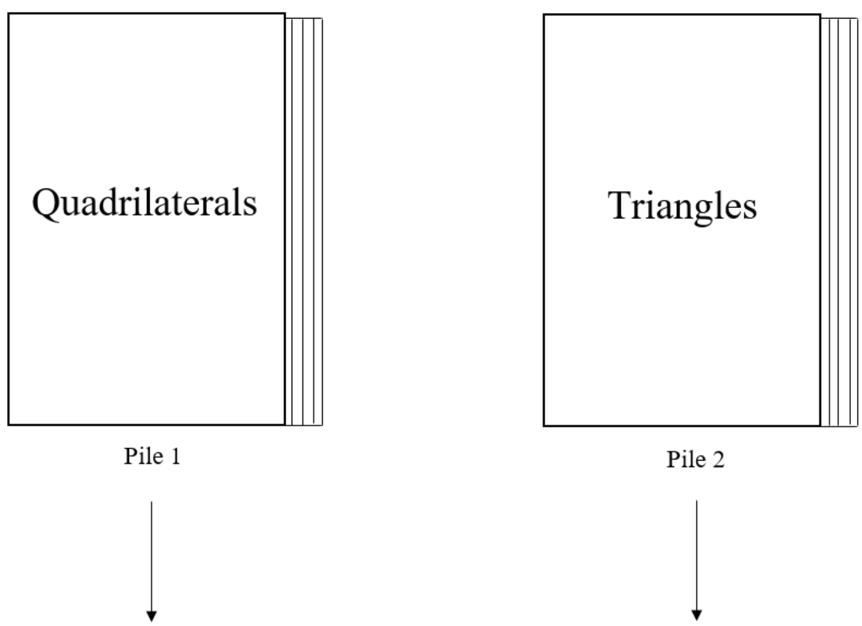
Research-Based Geometry Game

The geometry game consists of multiple piles of cards. Pile 1 is a deck consisting of cards pertaining to quadrilaterals. Each student draws five cards from this pile (unless the students opt for pile 2). Upon doing so, students may obtain many different classifications of quadrilaterals, such as trapezoids and parallelograms. Pile 2 consists of triangles. If a student chooses to draw five cards from the deck of triangles, they may receive isosceles triangles, equilateral triangles, right triangles, and so on.

Suppose a student's hand of cards is obtained from pile 1. Adjacent to pile 1 is another stack of cards which we will call pile 1.1. This stack consists of the different classifications of quadrilaterals. A student immediately draws a card from pile 1.1 subsequent to drawing form pile 1. If the card reads “trapezoid”, for example, then the student is to differentiate which of their five quadrilaterals is a trapezoid. If the student correctly identifies which of their quadrilaterals is a trapezoid, then they earn two points and move onto tier 2. Moreover, suppose the student’s hand does not contain any trapezoids. In this case, the student can declare that “zero of my cards have the indicated figure”. This student will still earn two points; however, they will not proceed to tier 2. Note that pile 2.1 will denote the classifications of triangles, e.g., right, equilateral, isosceles, and so on.

Tier 2 consists of rolling a six-sided die. The die is labeled one through six on each of its faces. Once a student has successfully identified the category of their figure(s) (and has earned two points), they must analyze their card(s). Continuing with the example of a trapezoid, suppose our student has two trapezoid cards in their hand. This hypothetical student may choose which card they would like to work with. The student will select one trapezoid card and discard the other back into the deck. The trapezoid card has various numbers at each side of the trapezoid, at each angle, etc. These numbers can range from one through six (just like the die). Consider the case where this student rolls a “2” on the die. As a result, the student is to reference their card and calculate the corresponding quantity designated by “2”. This can be an angle, a length of a side, or a command, such as “show an example of a rotation of the figure". If the student correctly answers the question or command signified by the designated number, they earn four points.

Finally, not all cards have number designations up to six. For instance, a single card may have a 1, 2, and 3 (each designating the angle measure at each vertex of a trapezoid, for example). What happens if a student rolls a 4, 5, or 6 on the die? In this case, the student must address a true/false question in a third pile of cards, labeled pile 3. The true\false question pertains to any topic in geometry such as tangent lines, fractals, rotations, translations, etc.



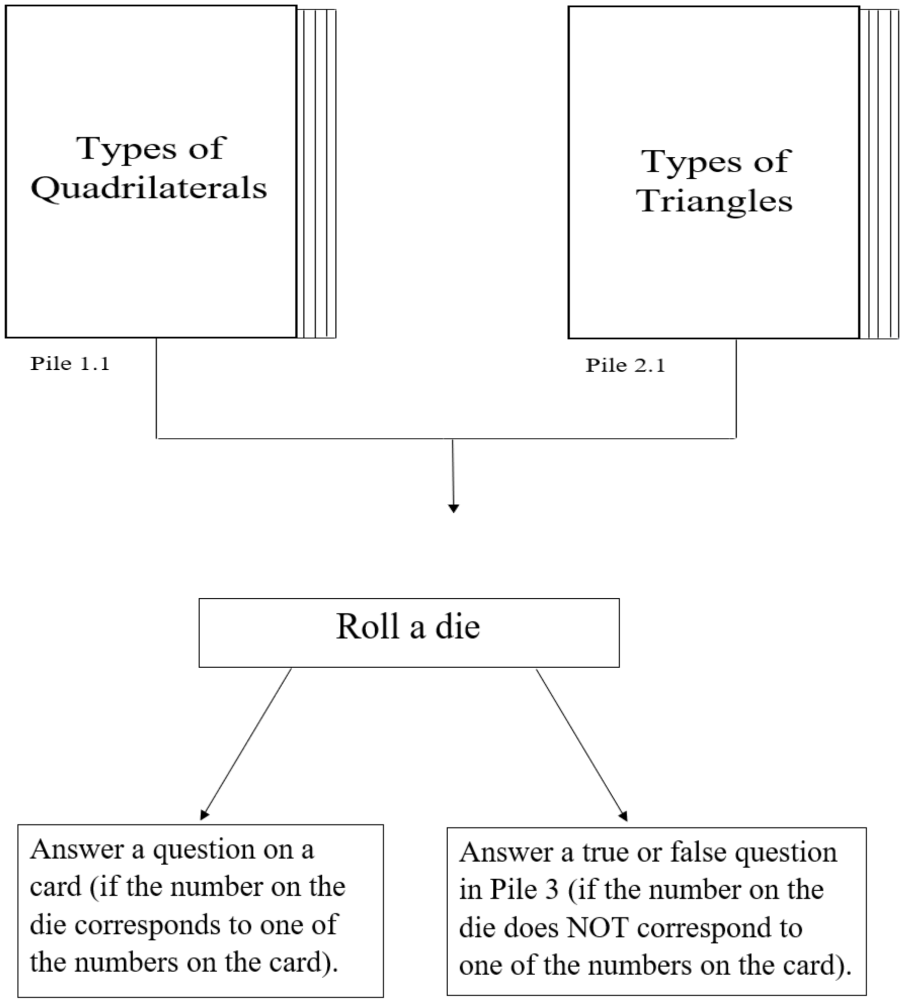


Fig. 1. Schematic of how the game proceeds

In the deck of cards, there will be many examples of the same shapes and diagrams. As a matter of fact, some problems will be exactly the same, aside from the numbers themselves for the lengths and the angle measures. This ensures repetition to reinforce the concept. According to Bright and Harvey (1988), “one of the strongest features of game playing is that a game presents students with many very similar problems that are therefore solved using the same problem-solving techniques” (p. 22). The figure below represents a hand of cards drawn:

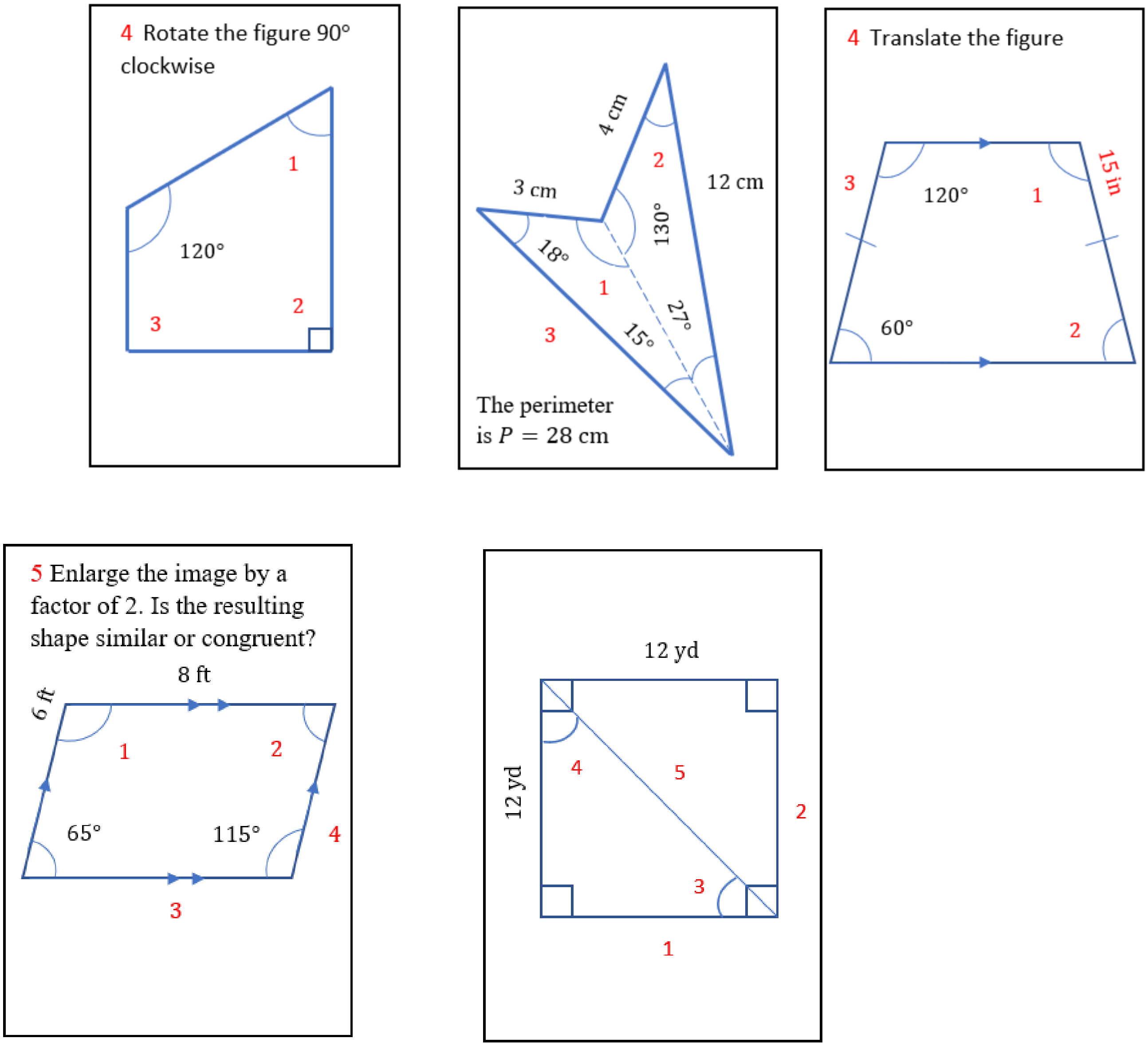


Fig. 2. A hand of five cards drawn by Pile 1 (quadrilateral pile)

Notice that some cards vary in difficulty. As a matter of fact, some may require the applications of theorems. Most of these cards are based off the generalization that, for any quadrilateral, the sum of interior angles is equal to 360°. For instance, the second card shows a quadrilateral represented as two triangles. The sum of interior angles of each triangle is equal to 180°. “Geometry games can accomplish a number of desirable instructional goals. For instance, they can furnish settings in which students visualize or construct geometric figures, see specific examples of general results or theorems, and apply logical reasoning skills in informal situations" (Bright & Harvey, 1988, p. 22).

What is the purpose behind Pile 3? Pile 3 consists of true-false and free-response questions. This allows students to participate in communicating with mathematical language. According to Bright and Harvey (1988) in *Games, geometry, and teaching*, the classification of quadrilaterals is usually taught through definitions (which they call a comprehension-level activity) and through grouping shapes based on commonalities (an analysis-level activity). Pile 3 allows one to achieve the instructional objective of comprehension-level activities mentioned by Bright and Harvey. For instance, the definition of a quadrilateral will be read to students and they can determine whether the given statement is true or false. On the other hand, Pile 2 engages students in analysis-level activity, forcing them to analyze which polygons are classified as parallelograms, for example. In this case, each of these polygons would be quadrilaterals consisting of two pairs of congruent (and parallel) sides. The following image depicts examples of cards in Pile 3 that use comprehension-level activity (since they are focused on defining the essential features) (Bright & Harvey, 1988). As a matter of fact, some show analysis-level activity as well.

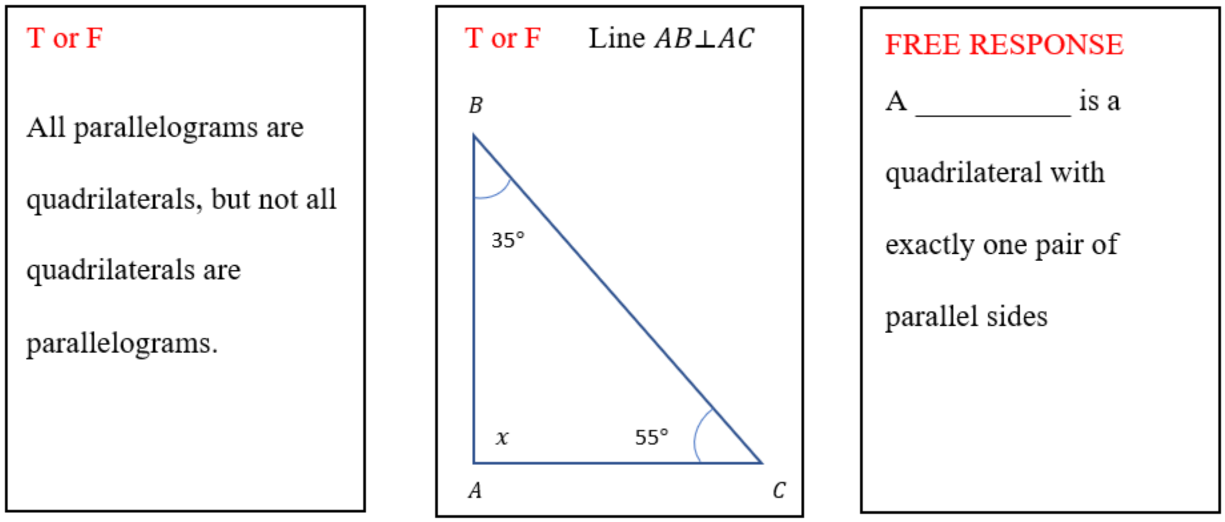


Fig. 3. Examples of cards that constitute Pile 3.

Notice that cards 1 and 3 focus on comprehension-level, whereas the second card actually focuses on the analysis-level. This is because the student has to make a logical leap, deducing that all right angles are congruent and have an angle measure of 90°.

Thus, $AB \perp AC$ iff $x = 90^{\circ}$.

Since the sum of interior angles in a triangle must be equal to 180°:

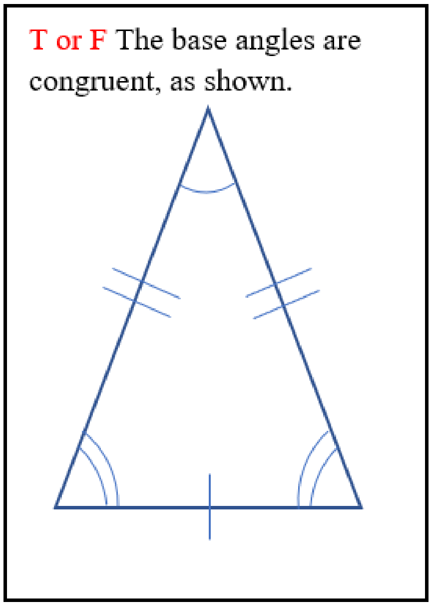
$$x + 35^{\circ} + 55^{\circ} = 180^{\circ}$$

$$x + 90^{\circ} = 180^{\circ}$$

$$x = 90^{\circ}.$$

$$\therefore\ AB \perp AC.$$

The next two examples of cards are also wonderful ways to illustrate the properties of parallelism and perpendicularity to students:



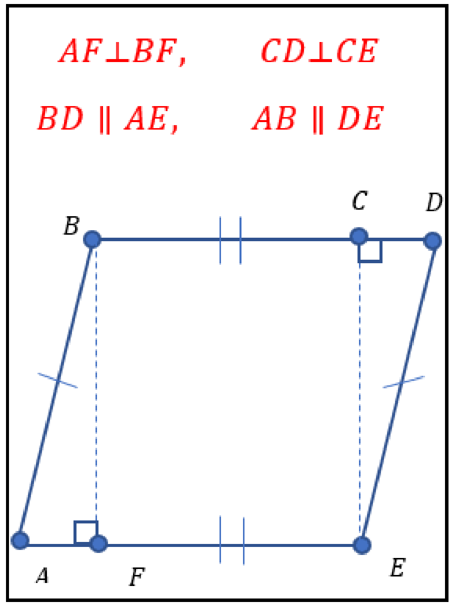


Fig. 4. Examples of an isosceles triangle and parallelogram.

In addition, the game can be extended even further by integrating technology. A study implemented by Turk and Akyuz (2016) indicated that the implementation of dynamic geometry systems (DGS) had a positive impact on student achievement and attitude in an eighth-grade geometry course. Students emphasized how the ability to draw and drag figures assisted them with comprehension (Turk & Akyuz, 2016). Since students commented on the interactive nature of DGS, the activity proposed in this paper can be extended via GeoGebra. In particular, one of the instructional objectives of this game is to teach transformations of geometric figures; hence, a dynamic software is ideal for such a lesson plan.

Furthermore, Turk and Akyuz (2016) prescribed a geometry achievement test (GAT) and geometry attitude scale (GAS) following the dynamic computer-based lesson. The results showed that students had an improved attitude towards geometry when they studied triangles using DGS. This reinforced the desire for the implementation of GeoGebra in this paper.

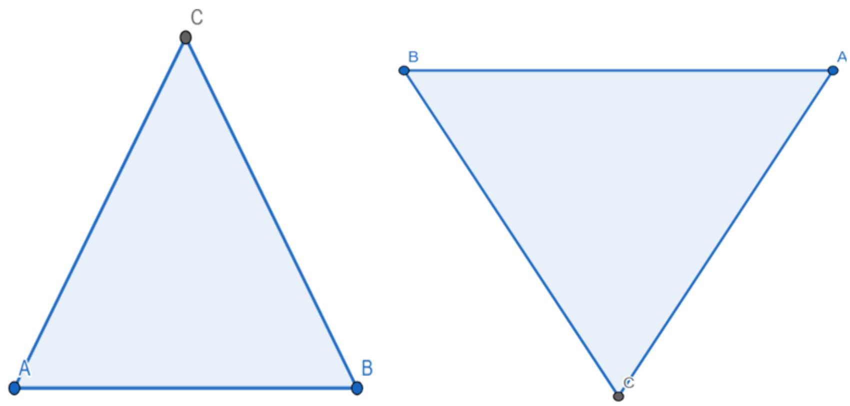


Fig. 5. The rotation of a triangle by 180 degrees using GeoGebra.

This allows students to easily demonstrate transformations on the computer rather than a dry erase board. This is also great for enlarging or reducing the size of figures.

Turk and Akyuz (2016) asserted that students even described the lesson as fun and meaningful when they manipulated triangles on GeoGebra to investigate the relationships between sides and angles. “Some of the students of the treatment group stated that their learning was easier and more meaningful than their past learning experiences since they explored topics themselves instead of memorizing formulas and theorems” (Turk & Akyuz, 2016, p. 101).

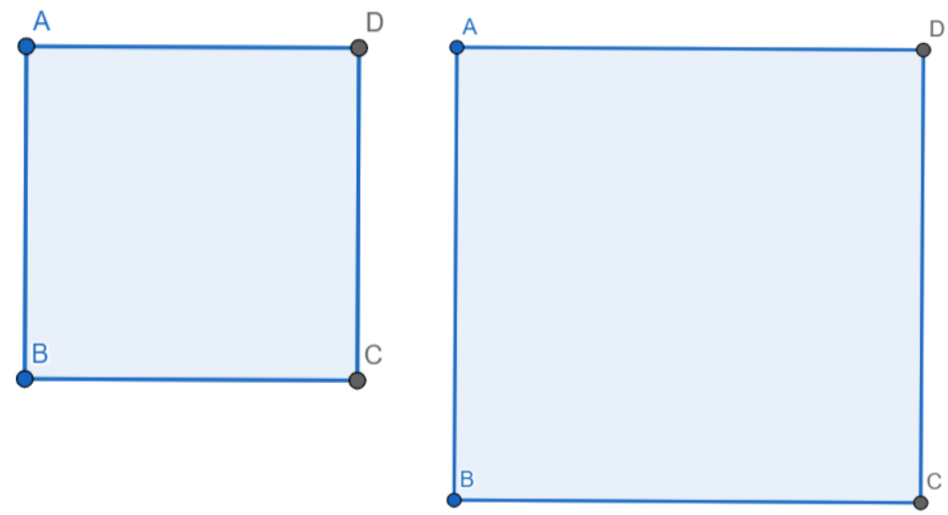


Fig. 6. Enlarging a square by some factor via GeoGebra.

Summary of Student-Teacher Experience and Evaluation

According to "Ideas for the Classroom" (2006), there was a mathematician named Gill Hatch (1937-2005) who was known for creating exciting algebra and geometry games for students. One of Hatch's works was a popular booklet called Jump To It! In this booklet, Hatch devises a series of games intended for students in the age group ranging from nine to 14 ("Ideas for the Classroom", 2006). "As Gill's work developed, Gill made it clear that she wanted to devise investigations and activities that were rooted in the curriculum and that developed the necessary techniques and skills alongside understanding and the ability to think mathematically" ("Ideas for the Classroom, 2006, p. 6).

Aside from the booklet for ages nine through 14, Hatch also developed an interesting geometry activity for older students. The game consisted of assigning a cylinder with a given volume to a group of students. Using the volume of the cylinder, students were asked to devise a creative way to determine the minimum and maximum surface area of that cylinder ("Ideas for the Classroom", 2006). According to "Ideas for the classroom" (2006), students had the experience of finding the minimum and maximum surface areas via *Excel* or via the application of calculus.

After observing students playing games for years, Gill Hatch asserts that the teacher must be actively present throughout the geometry game ("Ideas for the Classroom", 2006). When games are implemented as a pedagogic device, a teacher must have a deeper analysis of students' behavior in order to observe interesting outcomes, facilitate the game, and organize the game to where preparation does not consume class time ("Ideas for the Classroom", 2006). Hatch observed the following outcomes while incorporating geometry games in class: students received additional practice, pupils had a context for mathematical reasoning, individuals were welcomed to make generalizations, students were introduced to challenging ideas and situations, and children were supported by teachers and peers to work above their normal level ("Ideas for the Classroom", 2006).

Moreover, Hatch devised a highly interactive game called I-Like: Quadrilaterals. The strength behind this geometry game is that students work as one whole group alongside the teacher. The teacher shows a collection of quadrilaterals to the class (labeled one through ten, for example). With a specific property in mind, the instructor states “I like quadrilateral number six”, for instance. Based on this statement, students ask the teacher “do you like quadrilateral number nine?” Based on the teacher’s response, the students attempt to predict the property of the quadrilateral that the teacher has in mind (“Ideas for the Classroom”, 2006).

Bower (2004) provides a brief synopsis of Lynette Long’s book “Groovy Geometry: Games and Activities that Make Math Easy and Fun”. In addition to Bower’s synopsis, she describes her experiences with incorporating the book’s games and activities in the classroom. According to Bower (2004), her students were inclined to a particular game where the objective is to design a storybook. In “Shape Storybook”, students were expected to construct a polygon with more than four sides (Bower, 2004). As a matter of fact, the various games and activities can be adapted into a project as a follow-up activity (Bower, 2004). Bower (2004) describes her experience by commenting “my students enjoyed completing their project, and I enjoyed the students’ creativity” (p. 352).

Furthermore, another game in the book is called “How Tall”. This activity allowed Bower’s students to see applications of similar triangles. The objective of the game is to predict the height of an object using concepts of similarity (Bower, 2004). Similar to the previous game, this activity can also be adapted into the form of a project or incorporated into any geometry lesson. “Groovy Geometry includes valuable resource activities for hands-on discovery of geometric concepts. This book would be an excellent addition to any mathematics classroom" (Bower, 2006, p. 352). It is important to note, however, that each game has a section stating the required prerequisite material, such as terminology (Bower, 2004). Hence, when designing a geometry game, it is important to consider whether topics need to be reviewed before introducing them to students.

Conclusion and Suggestions

As stated by Bower (2004), many mathematics games can be applied in the classroom, adapted into follow-up activities, or even used as projects. It is important to incorporate these games in any form within a lesson plan. Furthermore, the synopsis of the articles suggests that games are successfully implemented when the teacher has a strong role as a participant. When designing a game, consider an activity similar to Gill Hatch's "I Like: Quadrilaterals", where the students must interact with the teacher, as proposed by "Ideas for the Classroom" (2006).

Similarly, not only do games provide excellent means for a teacher to participate, but they allow for the teacher to analyze their students. This provides the opportunity for the teacher to facilitate the game and observe ("Ideas for the Classroom", 2006). Additionally, designing a game has the practical purpose of implementing a point-system to quantify student-achievement. Bright and Harvey (1988) proposed that teacher-observation and point-systems during mathematics games serve as tools for informal assessment. Following the game, designing a corresponding project or follow-up activity will serve as a formal assessment. Therefore, it is suggested that a game has a point-system that accurately measures students' correct responses, teachers should have an active role in the game, and a formal assessment should still be provided (the game itself is not sufficient as a formal assessment; rather, an informal measure).

Finally, a game's strength is exponentiated when technology is incorporated. Similar to our paper *Fun with Measurements*, Gill Hatch used *Excel* in a game where students predicted the surface area of cylinders ("Ideas for the Classroom", 2006). Thus, when a game consists of measurements with many data values (such as cylinders of varying radii), *Excel* may be an excellent tool to teach students graphing data. Last but not least, another excellent tool to consider while designing a mathematics game is GeoGebra. Similar to Turk and Akyuz (2016), this paper demonstrates how GeoGebra is utilized to demonstrate transformations of plane figures. This allows students to visualize rotations, translations, and the scaling of geometric figures.

Although these recommendations are paramount to the design of a mathematics game, it is important to remember that these ideas can be extended to any mathematics course. It is strongly encouraged to incorporate similar games in algebra courses, for example. As a matter of fact, the game "I Like: Quadrilaterals" could be modified into a similar game called “I Like: Polynomials”, for instance. Perhaps the objective of this game would allow students to predict whether a teacher is describing a quadratic, cubic, or quartic function, etc. based off the properties of its graph. Extending this idea even further, students can graph the polynomial using *Excel* or GeoGebra and analyze its transformations, observing how the parent graph changes as the values of the leading coefficient is adjusted, and so on. Not only the possibilities are limitless, but the students will thoroughly enjoy themselves.

Lesson Plan (Include Technology to reinforce game)

<table>
<tr><td>Geometry Topic:

Perpendicularity, parallelism, tangency, congruency, similarity, and transformations (reflections, translations, rotations, scaling, etc.) for exemplified with quadrilaterals and triangles.</td><td>Grade Level & Course for Lesson:

The grade level for this geometry game pertains to grades eight through ten.</td></tr>
<tr><td>Learning Objective(s):

The instructional objective is to classify various polygons as quadrilaterals.
Students are expected to distinguish between parallelograms and trapezoids using concepts of parallelism.
Similarly, using the concept of perpendicularity, students can differentiate between a rhombus and a square.</td><td>Essential Question:

How can we classify and describe trapezoids, parallelograms, and triangles? What are the distinguishing features of a rhombus or a rectangle?</td></tr>
</table>

<table>
<tr><td>Content Standards (Focus):

1. CCSS.MATH.CONTENT.HSG.CO.A.1
2. CCSS.MATH.CONTENT.HSG.CO.A.2
3. CCSS.MATH.CONTENT.HSG.CO.A.3
4. CCSS.MATH.CONTENT.HSG.CO.C.10</td><td>Standards for Mathematical Practice:

1. Know the definition of perpendicular line and parallel line
2. Represent transformations in the plane. Compare transformations that preserve angle and distance versus those that do not.
3. Transformations of parallelograms, trapezoids, and regular polygons
4. Base angles of an isosceles triangle are congruent, the sum of interior angles in a triangle is equal to 180 degrees, etc.</td></tr>
<tr><td colspan="2">Estimated Time Frame:

This game is based on an activity by Bright and Harvey (1988) called Polygon Rummy. Bright and Harvey suggested 20 minutes for students to construct polygons using various sides and angles. Since our activity involves classifying and performing calculations, we suggest a duration of 30-45 min per session. Furthermore, our deck consists of multiple piles and more materials are required, in general for this activity.</td></tr>
<tr><td colspan="2">Prior Knowledge (Coherence):

• Students are expected to have a general understanding in complementary and supplementary angles, since this property is used to find missing angles inside various triangles and quadrilaterals.
• Students should have a strong foundation in the Pythagorean theorem, since it is utilized to calculate the length of a diagonal.
• Students should be familiar with vocabulary such as “base”, “leg”, and so on.</td></tr>
<tr><td colspan="2">Assessment/Performance Task:

The informal assessment consists of teacher-observation during the geometry game session. Moreover, the quantification of points provides another form of assessment. The formal assessment, however, consists of a worksheet done in class and a follow-up activity.</td></tr>
<tr><td colspan="2">Formative Assessment:

1. Triangles and Quadrilaterals: Trapezoids and Parallelograms Worksheet:

The purpose of this assessment is to measure students’ knowledge in perpendicularity and parallelism, extending those ideas to aid in the classification and distinction of various geometric shapes. Furthermore, students are exposed to translations, reflections, rotations, etc. The instructor will informally assess the students’ understanding using a point-system and teacher-observation.

• What classifies a polygon as a quadrilateral
• What classifies a quadrilateral as a trapezoid (describing the nature and number of its parallel sides)</td></tr>
</table>

- What classifies a quadrilateral as a parallelogram (describing its number of parallel sides)
- What is the altitude of a triangle? Is it always perpendicular to the base of the triangle?
- Do all right triangles contain two perpendicular line segments?
- Identify whether the quadrilateral or triangle experienced a rotation, reflection, etc. Upon transformations of two congruent figures, are they still said to be congruent? Are lengths and angles preserved? What if the image is enlarged by some factor? Are the angles still congruent?

2. **Discussion**

- Do parallel lines ever intersect?
- What shape is a parallelogram with 4 congruent sides and 4 right angles?
- What is the difference between a square and a rhombus?
- If a triangle contains two perpendicular legs, what type of triangle is this?

3. **Homework Assignment (Follow-up Activity)**

Please see the handout at the bottom.

Teacher preparation/Material List:

The following list is the set of required materials for this activity**:**

- Polygon playing cards
- 6-sided die (numbered 1 through 6)
- Dry erase boards for each student (to perform calculations and draw shapes)

Hands-on Manipulatives:	**Technology Tools/ Websites:**
Dry erase markers and small dry erase boards.	Computer and access to Geogebra website

Lesson Description:

1. The instructor will assign a student to be the card dealer. This geometry game has a deck of cards divided into many separate piles. As mentioned in the directions, the first pile of cards consists of questions pertaining to quadrilaterals, whereas the second pile pertains to triangles. The next two piles 1.1 and 2.1 consist of the different categories of quadrilaterals and triangles. Finally, one stack of cards entirely consists of true-false and free-response questions. **(5 MINUTES)**

2. Students will complete the warmup at the beginning of class prior to the PowerPoint presentation **(5 MINUTES)**.

3. After completion of the warmup activity, students will each receive 5 cards from the designated card dealer. Upon receiving their first five cards, they must draw from either 1.1 or

2.1. Each student is given roughly 10 seconds to make a decision. This decision is regarding which cards in their hand fit the category specified by the card drawn in piles 1.1 or 2.1. For instance, if 1.1 reads "trapezoid", this suggests that the students must determine which of the 5 cards in their hand has a trapezoid. For a class consisting of 25 students, for example, this would take about 4-5 minutes. **(5 MINUTES)** 4. The game is now fully in session. Students that correctly categorized the cards in hand will proceed to the next round. They draw a card that involves a calculation at this point, unless the die rolled results in a number that is not available on the card. If so, they must draw a card from the true-false or free-response pile. Since this step involves calculations, students may require more time on this portion of the game. **(30 minutes)** 5. The game is now completed. Student-achievement during the game is assessed, informally, via teacher-observation and the point-system. The last 15 minutes of class is dedicated to the formal assessment. A worksheet is administered to the students. **(15 minutes)**
Teacher Notes: Review the definition of a polygon before starting the game session. Additionally, discuss various transformations of geometric figures, such as reflections, rotations, translations, etc. Be sure to observe the class and actively participate. According to the research-based activities, it is important to analyze how your students respond to the game.
Application: Students will learn to describe features of plane figures using vocabulary such as "perpendicular" and "parallel". These words are important for the classification and description of kites, parallelograms, and trapezoids.

Differentiation/Accommodations		
ESE: **Numerous accommodations will be made for ESE students:** ● **Working with a partner** ● **Practicing the game beforehand, that way the student is comfortable with following the instructions real-time**	**ESOL/WIDA:** The instructor can assist students in reading the questions displayed on the card, writing the question on the board, or phrasing the question in a different way.	**Enrichment:** To ensure that students are challenged and engaged, the instructor can ask the student an EXTENSION of the question provided on the card. This is a hypothetical scenario that the card does not ask. For instance, after the student provides a correct response, the teacher can say "very good, would that statement also hold true for an equilateral triangle? Why do you think so?"

Formal Assessment (Worksheet)

1) **State true or false**

   a) **Each angle of rectangle is a right angle.**

   b) **The opposite sides of a trapezium are parallel**

   c) **The opposite sides of a rectangle are equal in length**

   d) **All the sides of a rhombus are of equal length**

   e) **All the sides of a parallelogram are of equal length.**

   f) **A triangle has three sides**

   g) **A triangle may have four vertices**

   h) **Every right triangle is scalene**

   i) **Each acute triangle is equilateral**

   j) **No isosceles triangle is obtuse**

Answer

a. **True**

b. **False**

c. **True**

d. **True**

e. **False**

f. **True**

g. **False**

h. **False**

i. **False,**

j. **True**

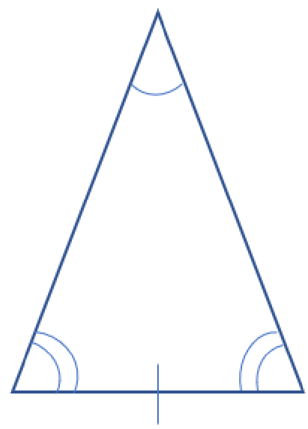
Follow-Up Activity

Polygons Worksheet: Quadrilaterals and Triangles

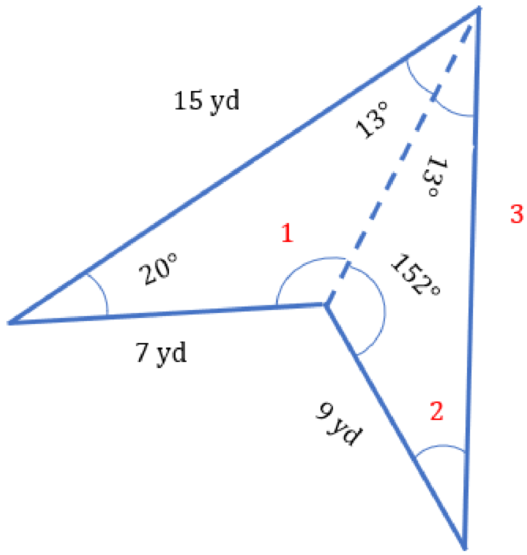
Name:___________________________

Date:____________________________

1. In the figure shown below, indicate whether or not the legs of the triangle are congruent.

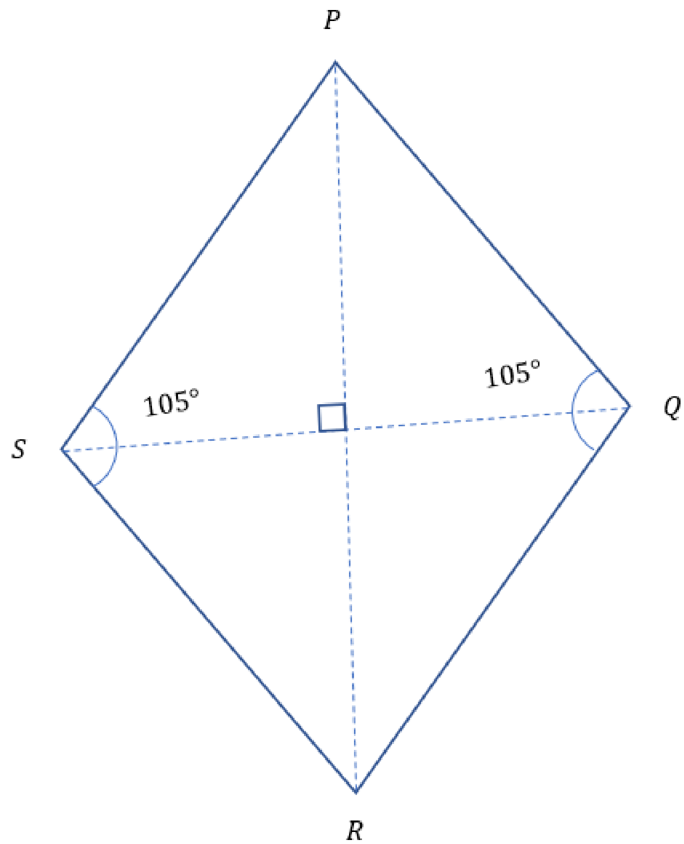


2. The following image depicts a quadrilateral with a perimeter of 15 ft. Using the given angle measures and the value of the perimeter, find the missing sides and angles.



$P = 47$ yd

3. The figure below depicts rhombus $PQRS$.



(a) If each side of the rhombus was perpendicular to one another, then each interior angle would have a measure of 90°. What shape would rhombus $PQRS$ be?

(b) What is the measure of $\angle SPQ$?

(c) State whether or not the following statement is true:

The diagonals are perpendicular ($PR \perp SQ$)

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