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Solving Logic Problems with a Twist

Cover Page Footnote

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Solving Logic Problems with a Twist

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Review of Symbolic Logic

Aristotle, Greek philosopher and scientist, was a pupil of Plato and asserted that any logical argument is reducible to two premises and a conclusion (Gullberg, 1997). According to classical (Aristotelian) logic, arguments are governed by three fundamental laws: the principle of identity, the principle of the excluded middle, and the principle of contradiction. The first law states that *a thing is itself*, i.e., $A = A$. According to the principle of the excluded middle, a proposition is prescribed a discrete truth value (true or false, but there exists no value in between). Lastly, the principle of contradiction asserts that a *single* truth value can be assigned to a statement. A proposition must either be true or false, A or *not* A (Gullberg, 1997).

The intimate relationship between logic and mathematics was established by German mathematician and philosopher G.W. von Leibniz (Gullberg, 1997). Leibniz is recognized as the founder of symbolic logic and for his discovery of a *lingua universalis* – a language where logical errors would be equivalent to errors in mathematics. Furthermore, English mathematician and logician George Boole published the *Mathematical Analysis of Logic* (1847) (Gullberg, 1997). In this work, Boole devised a system of algebra to express logical relations. In 1854, Boole expanded on his algebra in *An Investigation into the Laws of Thought, on Which are Founded the Mathematical Theories of Logic and Probabilities*, eventually resulting in the extension of Boolean algebra into electronics (applications in digital circuits) (Gullberg, 1997). Finally, Gottlob Frege is acknowledged as the founder of modern symbolic logic (Gullberg, 1997). In 1879, his contributions to predicate logic were acknowledged in his publication *Begriffsschrift: Eine der aritmetischen nachgebildete Formelsprache des reinen Denkens* (“Concept-Script: A Formal Language of Pure Thought on the Pattern of Arithmetic.”) (Gullberg, 1997).

Propositions are statements that can be assigned truth values. As mentioned previously, truth values constitute a binary system: the two possible truth values are T or F . In the context of Boolean algebra, these truth values may be referred to as Boolean constants, where 1 denotes true and 0 denotes false. A tilde \sim is applied in order to negate a proposition; hence, $\sim p$ is read “not p ”. The variable p represents any arbitrary proposition. For instance, suppose we let p denote “the ball is red”. Negating this statement results in “the ball is not red”, or $\sim p$. Note that “is red” is referred to as the predicate, i.e., the portion of the statement describing the state or condition of the noun.

In addition to negation, another logic symbol is the universal quantifier, \forall , denoting “for every” or “for all” (Gullberg, 1997). As an example of its application, suppose you wish to state that a polynomial $P(x)$ is defined for all real numbers, i.e., its domain is $(-\infty, \infty)$. An equivalent symbolic representation is provided by $P(x)$ is defined $\forall x \in \mathbb{R}$ (“ $P(x)$ is defined for every x belonging to the set of real numbers”). Similarly, the existential quantifier \exists also describes *how*

much or how many values can belong to a set. For instance, $\exists x \in \mathbb{R}, 2x - 5 > 7$ (“there exists at least one x belonging to the set of real numbers, such that the proposition (inequality) is valid”) (Gullberg, 1997). Moreover, the conjunction \wedge denotes “and” and is analogous to multiplication in Boolean algebra. For instance, in symbolic logic $A \wedge B$ reads “A and B”, and is denoted as $A \otimes B$, $A \cdot B$, or simply AB in Boolean algebra. Furthermore, the disjunction \vee indicates “or”. The proposition $A \vee B$ reads “A or B”, and is denoted $A \oplus B$, or simply $A + B$ in Boolean algebra. Note that subtraction does not exist in logic; therefore, division does not exist (since it is repeated subtraction). On the other hand, because multiplication is compounded addition...

$$x \oplus x = x \quad \text{and} \quad x \otimes x = x.$$

These properties will be discussed in the following sections.

Furthermore, propositions can take the form of “if-then” statements: $A \Rightarrow B$, read “A implies B” or “if A then B”. In this logical expression, the term preceding the implication symbol is referred to as the antecedent (hypothesis), whereas the subsequent term is the conclusion (Gullberg, 1997). Note that the expression $A \Rightarrow B$ is false *only* when A is true and B is false (Gullberg, 1997). We have discussed the procedure for combining quantifiers or unary connectives to propositions (operators that have a single statement as their operand, such as \forall, \exists, \sim), combining two propositions via binary logical connectives (such as \oplus, \Rightarrow , etc.), and now we can apply this foundational knowledge to form even larger logical expressions consisting of many logical connectives. For instance, we can form expressions such as

$\sim A \Rightarrow \sim B$, which contains two unary connectives (the negations acting on each term) and a binary connective (the implication acts on both $\sim A$ and $\sim B$). Suppose A represents the proposition “two lines intersect at point, P” and B denotes “the ordered pair describing P is a solution to a system of equations”. This suggests that the statement $A \Rightarrow B$ indicates “if two lines intersect at point P, then the ordered pair describing P is a solution to a system of equations”. The inverse of this statement is found by negating both sides (and it is logically equivalent). In other words, $\sim A \Rightarrow \sim B$ has the same meaning as $A \Rightarrow B$.

Finally, these complex propositions can constitute the premises and conclusions of our logical arguments. Now that we are familiar with the notations, we can practice constructing a few basic arguments. According to Palmer (2011), an argument is *valid* if the premises logically entail the conclusion. In other words, there is no flaw in the *form* of the argument or its reasoning. A common logical argument takes the form:

1. $A \Rightarrow B$
 2. A
- $\therefore B$

A logical argument of this form is called *modus ponendo ponens* (Palmer, 2011). As a concrete example:

1. **If I choose door 1 then I am free**
 2. **I choose door 1**
- \therefore I am free.**

An inference of the form:

1. $A \Rightarrow B$
 2. $\sim B$
- $\therefore \sim A$

is called *modus tollendo tollens* (Palmer, 2011). Notice how the conclusion in premise 1 is negated in premise 2. The conclusion of the argument suggests that the antecedent is negated.

Finally, it is important to discuss the meaning of a *tautology*. “A tautology is a proposition in which the clause following the word “is” or the word “are” (i.e., what is called in grammar the copula) repeats in some form or other all or part of the clause before the copula”(Palmer, 2011). All equations and definitions are tautological since they state that two expressions of different forms are equal. For instance, “a sister is a female sibling”, “a fork is a utensil”, “ $x^2 - 4 = (x + 2)(x - 2)$ ”. The statement “a derivative is the slope of a tangent line to a point on a curve” is not a tautological proposition. This is because information is not repeated in the expression equated to “derivative”; rather, additional information is provided.

Five Additional Logic Problem Examples and Applications

1. Digital Circuits – Finding Boolean Output Expression for Logic Gate

The following truth table depicts three inputs A , B , and C and an output X for a digital circuit in an electronic device. A zero (0) denotes a component that is switched “off” in the circuit, whereas a one (1) indicates that an element is switched “on”. Find (a) the logic expression for the output, X , (b) a simplified logic expression for the output X , (c) design a simpler circuit, and (d) construct a Karnaugh map (K-map).

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
1	0	0	0
0 \bar{A}	1 B	1 C	1
1 A	0 \bar{B}	1 C	1
1	1	0	0
1 A	1 B	1 C	1

(a) Boolean expression for output, X

The output occurs when it has a truth value of 1; therefore, each term in our expression will constitute a row in the truth table that satisfies this condition. Referring to the given truth table (above), start by circling each 1 in the column for the output, X . In each row, if an input variable is assigned a truth value of 0, then this indicates a negated input. For instance, if A is designated a truth value of 0, then our expression for the output states, \bar{A} .

Therefore, the output is given by...

$$X = \bar{A}BC \oplus A\bar{B}C \oplus ABC.$$

(b) Simplify the expression for the output

$$X = \bar{A}BC \oplus A\bar{B}C \oplus ABC$$

$$= \bar{A}BC \oplus AC(\bar{B} + B)$$

$$= \bar{A}BC \oplus AC$$

$$= (\bar{A}B \oplus A)C$$

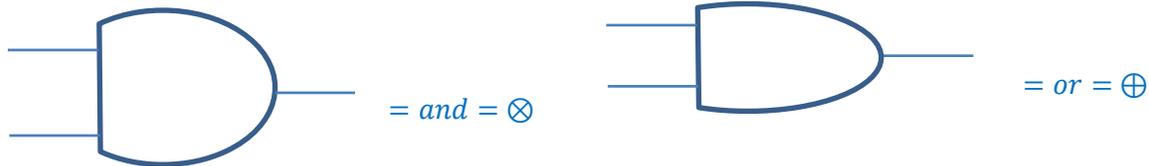
$$= (A \oplus B)C.$$

Since AC appears in both term 2 and term 3, we can factor out this expression. Although tempting, do not combine any like terms in the manner: $A + A = 2A$, for example. Recall, this is Boolean algebra, not traditional algebra. The notation \oplus serves as a reminder.

Note: Line 2 was simplified via the property: $\bar{B} + B = 1$.
Line 4 was simplified via the property $\bar{A}B \oplus A = A \oplus B$.

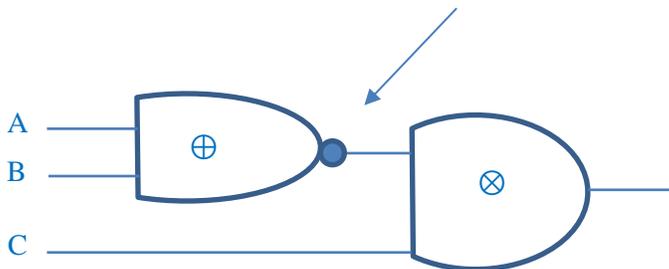
(c) Design a circuit based off the simplified expression for the output:

In the schematic, note that...



Therefore, the circuit corresponding to the output $X = (A \oplus B)C$ is...

At this point, your (binary) logical operation \oplus acts on inputs A and B , outputting $A \oplus B$.



This **gate** is also analogous to a **function machine**, where the inputs are $A \oplus B$ and C , the function (operation) is the \otimes , and the output is $(A \oplus B) \otimes C$.

(d) Construct a Karnaugh map:

In applications such as circuits and computer science, a Karnaugh map is a graphical representation of the simplified Boolean expression extracted from the truth table. Using a graphical method as an alternative solution is a wonderful problem-solving strategy, because it verifies that our logical (Boolean) expression for the output is correct. To construct a Karnaugh map, reference the given truth table and count the number of rows. Since the truth table consists of 8 rows, our Karnaugh map is comprised of 2 columns and 4 rows. Each entry in the Karnaugh map is an output (the value for X) corresponding to each input.

		C	
A	B	0	1
0	0	0	0
0	1	0	1
1	1	0	1
1	0	0	1

The entries for the inputs, A and B , are arranged in a memorized pattern known as *gray code* (or reflected binary code) in computer science. Similar to a truth table, it is simply all possible combinations of ones and zeros. As mentioned, the entries in each cell denote a value of the output variable. For instance, the first cell reads “the output is $X = 0$ when $A = 0, B = 0$, and $C = 0$ in the truth table. In column 2, row 2 the Karnaugh map reads “ $X = 1$ when $A = 0, B = 1$, and $C = 1$ in the truth table.

Although the Karnaugh map is complete, let’s extract the simplified Boolean expression for the output from this graphical representation. Previously, we painstakingly derived this logical expression from the truth table itself. After doing so, we manipulated this complex logical expression and simplified it via properties of Boolean algebra. Utilizing our Karnaugh map, let’s verify that we obtain the *same* expression for X . The next step is to circle groups of 1’s because each term in the expression for the output corresponds to $X = 1$, otherwise the device is switched off. You can only form groups of 1 by circling entries horizontally or vertically (not diagonally).

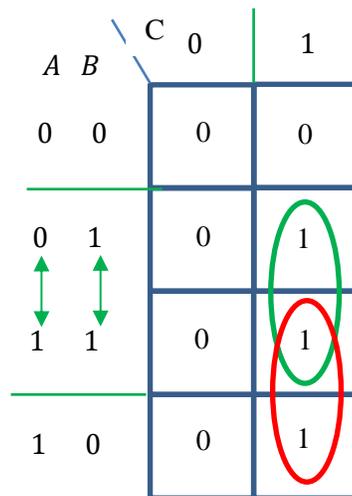
Furthermore, the objective is to enclose as many entries as possible in each group; otherwise, you will have more terms than necessary in your logical expression (potentially repeating or redundant terms). As a consequence, you would not derive the expression for output in its simplest form, which would require you to simplify it via Boolean algebra, defeating the purpose of this method. Again, the benefit of this method is that you find the output in its simplest form without having to do any algebra, otherwise it becomes the same procedure as finding the expression directly from the truth table itself. Another rule pertaining to grouping 1’s in the Karnaugh map is that you can only circle 2^n entries. For instance, a group can consist of $2^0 = 1$ ones, $2^1 = 2$ ones, $2^2 = 4$ ones, $2^3 = 8$ ones, and so on.

		C	
A	B	0	1
0	0	0	0
0	1	0	1



Notice how there are two entries enclosed by the **green (upper) contour**. We cannot enclose three entries because it violates the rule that a group must contain 2^n elements. Moreover, notice how we could have enclosed one entry; however, our goal is to optimize the number of Boolean constants enclosed in a group. The next group (**red (lower) contour**) could also enclose a single element, but again we wish to maximize the number of members belonging to a set. Therefore, a property of the Karnaugh map is that we can **overlap** groups, such that members of one group can also belong to another set.

The final law governing Karnaugh maps is that if an input variable changes its value in the group's row or column, then it is not included in the output expression.



Rows where green group resides (rows 2 and 3): $A: 0 \rightarrow 1$
 $B: 1 \rightarrow 1$

Column where green group resides (column 2): $C: 1$

Therefore, since the input variable A alternates values, it represents terms such as $A \oplus \bar{A}$, which is equal to one. We do not include these additional variables in our expression because their value is

unity. On the other hand, B is constant in the rows of interest, i.e., it does not fluctuate between truth values; hence, it denotes an expression of the form $B \oplus B = B$. Note the benefit of the notation \oplus opposed to “+” is that it serves as a reminder that like terms are not combined the same way as they are in traditional algebra, where $B + B = 2B$. However, either notation can be utilized; it is simply a matter of preference. Again, this is because $\forall B \in \mathbb{R}, B + B = 2B$ in algebra, whereas $\forall B \in \{0, 1\}, B \oplus B = B$ in Boolean algebra. Finally, the green loop only occupies a single column, and therefore, C is constant at the value 1.

Putting this all together, we see that one term in the output must be CB .

Looking at the red loop, we do a similar procedure:

		C	
		0	1
A	B		
0	0	0	0
0	1	0	1
1	1	0	1
1	0	0	1

↕ ↕
↕ ↕

Rows where red group resides (rows 3 and 4): A: 1 → 1
B: 1 → 0

Column where red group resides (column 2): C: 1

Therefore, term two in the output is given by... CA.

Note that B is not included, because its value alternates (suggesting that its expression takes the form that is equal to 1). Finally, combining the term from the green group with the term in the red group gives the logic expression for the entire truth table. Now for the moment of truth...

$$X = CB \oplus CA = C(B \oplus A)$$

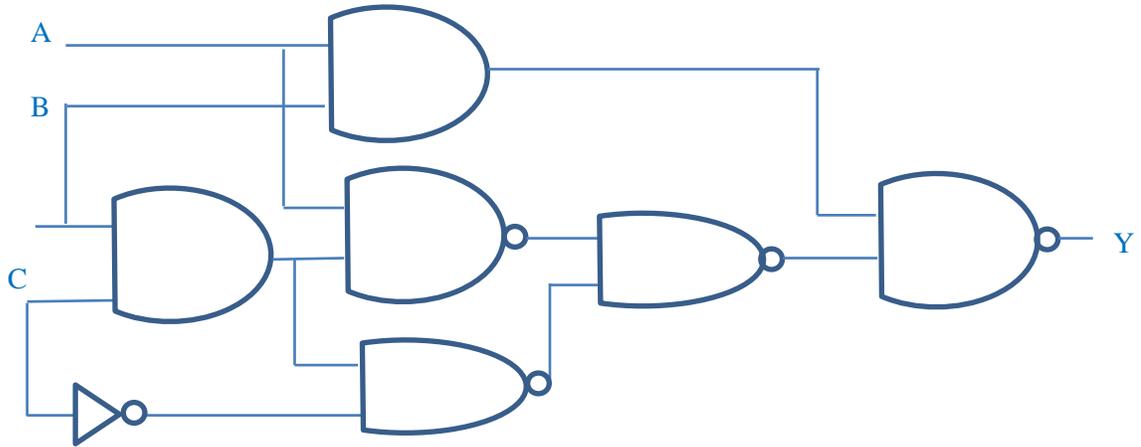


The proof is complete. The logical expression derived from the Karnaugh map is equivalent to the expression directly obtained from the truth table and brute force Boolean algebra.

2. Digital Circuits – Finding the Boolean Output Expression Directly from Logic Gates

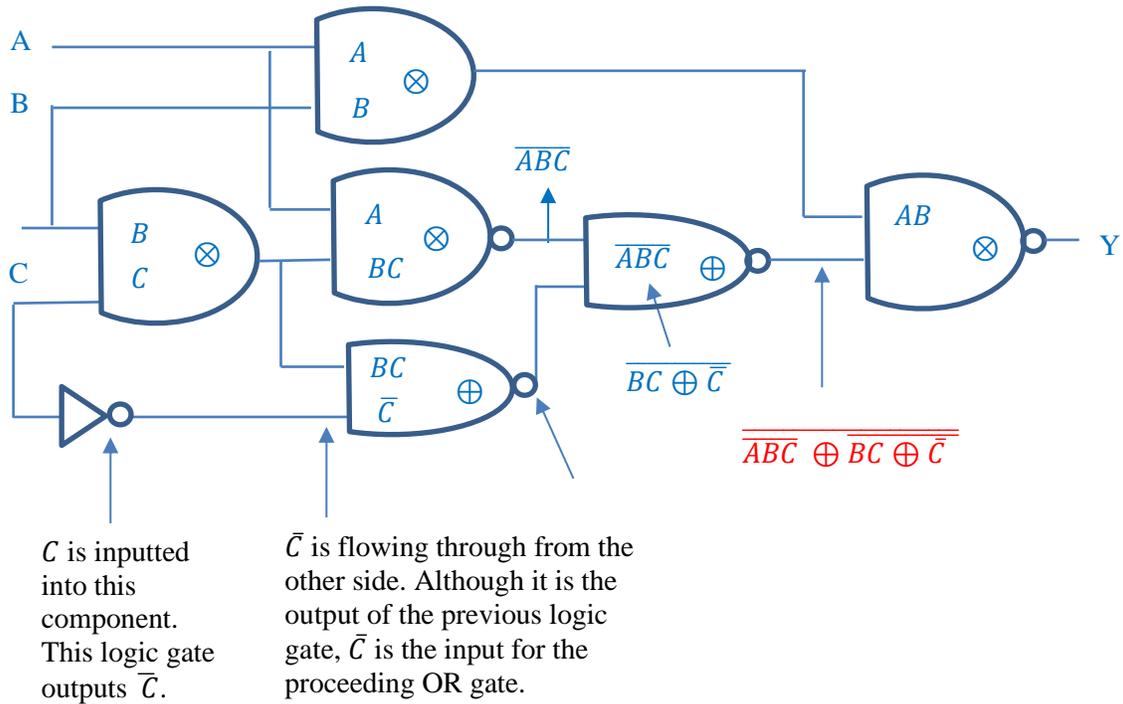
Suppose the following schematic represents an electronic device with inputs A , B , and C . Find the Boolean output Y corresponding to the logic gate diagram.

Note: After obtaining the logical expression, reduce it to a simpler expression using properties of Boolean algebra. This is a great exercise that also allows you to alternate between logic circuits, truth tables, and logic expressions.



As mentioned in the previous problem, each logic gate is analogous to a function machine, processing and operating on our inputs. The AND gate multiplies two inputs, whereas the OR gate adds two operands. A great strategy is to write “OR”, “AND”, “ \oplus ”, or “ \otimes ” on a respective schematic symbol, allowing you to navigate through the diagram with ease. Furthermore, some new symbols have been introduced in this new schematic:  is just another symbol for NOT and it can be tacked onto any other symbol in the diagram. The symbol  is the NOT gate and it simply negates an input value and does nothing more. If the input is A then this function machine outputs $\sim A$, or in the notation of Boolean logic, \bar{A} .

Notice as each input traverses the logic circuit; it receives additional processing (it is operated on more and more by various binary and unary operators). As you navigate through the circuit, the Boolean expression will become increasingly complex as it is outputted from logic gate to logic gate (function machine to function machine). Our goal is to ultimately determine the final product, or the output generated at the very end of the circuit. Inevitably, we will simplify this complex expression into a compact form. Assigning the (Boolean) logic symbol to each component in the schematic...



The expression is becoming rather complex. We could keep building our logical expression and simplify once it exits the final logic gate. Alternatively, we could have simplified the logical expression each step along the way. Before inputting the **red expression** into the final AND gate, let's simplify the sum...

$$\begin{aligned} & \overline{\overline{ABC} \oplus BC \oplus \bar{C}} \\ & = \overline{(\overline{ABC}) \otimes (BC \oplus \bar{C})} \end{aligned}$$



Applying **De Morgan's Theorem**: The bar covering the entire expression is split between two terms (where the first \oplus appears). The first \oplus then becomes \otimes .

At this point, there are numerous ways to simplify the logical statement. By inspecting the expression above, one can recognize the form $BC \oplus \bar{C} = B \oplus C$. Also notice, we applied the property $\bar{\bar{x}} = x$ (this has the same meaning as $\sim(\sim x) = x$). This illustrates the purpose of De Morgan's Theorem, allowing us to cancel some NOT operators...

$$\begin{aligned} & (ABC) \otimes (B \oplus C) \\ & = \overline{ABC}B \oplus \overline{ABC}C \\ & = ABC \oplus \overline{ABC} \\ & = ABC. \end{aligned}$$

Simplify according to the property

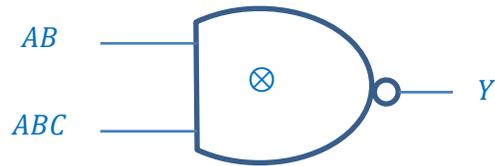


$BB = B$ and $CC = C$.



Apply the property $X + X = X$

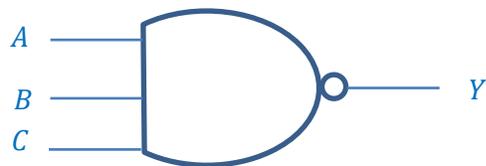
Therefore, $\overline{\overline{ABC} \oplus BC \oplus \bar{C}} = ABC$ is one of the inputs in the FINAL logic gate. Recall, the schematic representing the final logic gate was...



Performing the logical operation of the AND gate... $\overline{AB} \otimes \overline{ABC} = Y$

$$\overline{ABC} = Y.$$

The entire logical output of that complicated circuit was reduced to a single term, where each of the three input variables occur once in the expression. Likewise, the simplified schematic corresponding to the original circuit is...



In the first example, we were given a truth table and then we converted it into a complex logical statement. After doing so, we simplified the compound proposition into a simpler expression via Boolean algebra. Finally, we proved that the simple expression derived from algebraic manipulations was correct, via a graphical solution utilizing Karnaugh maps. In the second application problem, we saw how propositions are analogous to input variables. Each logic gate is analogous to a function machine, and as the input traverses the circuit, more operations are performed on these logical statements, until they become complex propositions. Our next few examples depict truth tables from pure discrete mathematics (outside the context of electronics and computer science). We focused on methods of *finding* expressions for logical statements (from truth table to logical statement or from logic gate to logical statement). In our next example, we are *given* a logical statement and required to determine its corresponding truth table.

3. Constructing a Truth Table from a Given Logical Statement

Suppose you are given the proposition, $p \rightarrow \sim(p \wedge q)$. Construct a truth table to model this given situation.

p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \rightarrow \sim(p \wedge q)$
T	T			

T	F			
F	T			
F	F			

We start by writing all possible combinations of T and F (this is analogous to the gray code in the computer science example: all possible combinations of 1's and 0's). The first two columns are the propositions without any logical connectives. Then, we depict the expression inside the parentheses in the next column, and gradually continue to work our way out until we reach the desired statement.

The statement $p \wedge q$ has a truth value of T only when *both* p and q are true. For instance, in row 3, q possesses a truth value of false; hence, the entire statement $p \wedge q$ is false. Filling in the third column results in...

p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \rightarrow \sim(p \wedge q)$
T	T	T	F	
T	F	F	T	
F	T	F	T	
F	F	F	T	

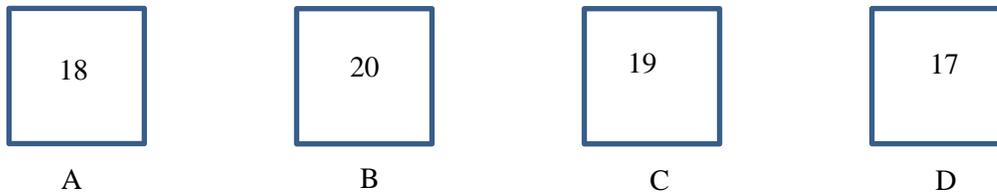
The next column negates our previous column. This led to the truth values in blue. Finally, the last column contains the implication arrow. Regardless of the truth value of the antecedent, if the conclusion is true, then the entire statement $p \rightarrow \sim(p \wedge q)$ is true. If the conclusion is false, then the entire statement is false. Comparing our first column to our blue column...

p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \rightarrow \sim(p \wedge q)$
T	T	T	F	F
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

The truth table is complete. The statement is false only in the first row of the truth table.

4. Using logic to answer quantitative problems

Let's assume there are 4 students in the classroom: students $A, B, C,$ and D . The teacher administers writing utensils so each student can complete their assignment. Each student in the class received; 4, 5, or 6 pencils. Suppose students that receive 4 or 6 pencils always tell the truth, whereas students administered 5 pencils never tell the truth. When asked to report the total number of pencils the group received, each student stated the following numbers:



Given that three of the four students are lying, i.e., they received 5 pencils, which student is the honest student?

There is only one honest student, since the given problem stated that three students received 5 pencils. This suggests that a total of $5(3) = 15$ pencils were distributed to dishonest students.

Subtracting 15 pencils from each student, we can determine which student received either 4 or 6:

$$A: 18 - 15 = 3$$

$$B: 20 - 15 = 5$$

$$C: 19 - 15 = 4$$

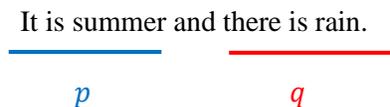
$$D: 17 - 15 = 2$$

Therefore, the only student with 4 or 6 pencils remaining, is student, C .

5. Suppose you are given the following statement: It is summer and there is rain.

- (a) Express the given statement symbolically
- (b) Apply De Morgan's Theorem to negate this statement
- (c) Construct a truth table for the statement expressed in part (b).

The given statement consists of two propositions p and q combined by a single connective (and).



- (a) Expressing the statement symbolically: $p \wedge q$
- (b) Negating the statement using De Morgan's Theorem: $\sim(p \wedge q) = \sim p \vee \sim q$

Note that we applied the same process in one of the application examples. In De Morgan's Theorem, the sign alternates from multiplication to addition or vice versa.

Hence, the statement reads "it is not summer or there is no rain".

(c) Devise a truth table for the statement $\sim p \vee \sim q$, i.e., “it is not summer or there is no rain”. Since there are 2 statements, our truth table will consist of $2^2 = 4$ rows:

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Solution to the Given Problem:

Suppose a prisoner must make a choice between two doors. One door leads to freedom and the other door is booby-trapped so that it leads to death. The doors are labeled as follows:

Door 1: This door leads to freedom and the other door leads to death.

Door 2: One of these doors leads to freedom and the other of these doors leads to death.

If exactly one of the signs is true, which door should the prisoner choose? Give reasons for your answer. Provide a detailed explanation of how you approach the problem. Use at least two different models for your solutions.

Solution (Model) 1.

Convert the verbal (written) statements to symbolic logic. In order to create a logical expression, we will start by assigning arbitrary variables to each proposition.

Let p denote *Door 1 leads to freedom* and let q denote *Door 2 leads to death*. Using this notation, the first proposition becomes...

$$p \wedge q$$

Statement two can also be expressed symbolically...

$$(p \wedge q) \vee \sim(p \wedge q)$$

Using De Morgan's theorem, we can replace $\sim(p \wedge q)$ with $\sim p \vee \sim q$...

$$(p \wedge q) \vee (\sim p \vee \sim q)$$

Perhaps this forms a complex logical expression with some redundancies. If this is the case, we can form a compound proposition, simplify it, and then translate the meaning of the compact logical statement. Combining propositions 1 and 2 gives...

$$(p \wedge q) \vee (p \wedge q) \vee (\sim p \vee \sim q).$$

This expression simplifies to... $p \wedge q$

Therefore, the compound proposition formed from both doors reduces to the statement on door 1. Thus, door 1 leads to freedom and door 2 leads to death.

Solution (Model) 2.

Door 1: This door leads to freedom and the other door leads to death.

Door 2: One of these doors leads to freedom and the other of these doors leads to death.

Door 1 is p and door 2 is q . Using the formula if p , then q , the only conclusion that makes the statement true is, if door 1 leads to freedom, then door 2 does not lead to freedom i.e. death.

p	q	$p \wedge \sim q [(p \wedge \sim q) \vee (\sim p \wedge q)]$
F	F	F
F	T	F
T	F	T
T	T	F

Summary

The crossroads of logic and mathematics breeds critically thinking students that have the confidence and independent working skills to tackle any problem they come across. In this artifact several applications of logic to common problems have been applied in a way that would allow teachers to use the examples in their classrooms in a wide range of grade levels. More specifically Boolean Algebra and Truth Tables have been making their way into middle and high school mathematics classrooms with the nations shift from just being able to compute to understanding on a cognitive level why you solved the problem the way you did. In United States schools the standardized testing is becoming increasingly rigorous as the years go by and mathematics concepts that used to be reserved for higher education students is now considered everyday math. “Mathematical knowledge and the ability to solve quantifiable problems and utilize critical thinking skills enhance the abilities of students to think and make decisions (Su, 2016). Applying logic to common math problems in low pressure classroom setting prepares students and interests them in higher level mathematics applications. The goal of every teacher is to expand the mind of and inspire their students to go farther in the subject matter than they ever thought possible and teaching them these logic skills and applications will help them get there.

Boolean Logic is a method of algebra that explores three words commonly known as Boolean Operators: “Or,” “And,” and “Not”. Boolean logic is at its core the basic idea that any and all statements are either true or false. The use of Boolean Logic allows students to think on a critical level of more complex mathematical applications, this allows students to categorize whatever information is given and use deductive and/or inductive reasoning skills to arrive at an answer to any problem. The use of simple “and”, “or,” and “not,” Venn diagram problems are found commonly in Algebra 1 classrooms and can be found as early as Pre-Algebra in some middle school classrooms. Teaching students the terminology and logic behind these types of problems gives them a concrete foundation to apply these skills in their higher-level mathematics classes. It is essentially giving them a foundation for their future probability courses and has many other applications. Introducing students to “out of the box” ways to approach simple problems with pique their interest and

encourage them research independently and be excited to share what they have learned with the class.

One of the tools of Boolean Algebra that was used in this artifact is a truth table. Truth tables are mathematical tables that organize the true or false outcomes to compound statements. Each statement is usually represented by a variable, most commonly p, q, and r, and each statement has its own corresponding column in the table that contains the likely truth values. Truth tables could be introduced in high school geometry when they are learning reasoning skills. “By using inductive and deductive reasoning and questioning, while adhering to the Blooms Taxonomy, students learn mathematical concepts and solve mathematical problems and also recognize the extent to which reasoning applies to mathematics and how their critical thinking pathway can facilitate decision making and strategic planning (Su, 2016).” Even though logic is considered higher order thinking and complex, when teachers introduce students to these concepts as soon as possible and constantly over time, they become simple. Promoting the highest levels of Bloom’s Taxonomy will enable teachers to ensure all students are mastering the content while building habits that will aid them in their chosen career.

Use in the Classroom

Teachers are under incredible pressure to teach content and prepare students for standardized tests, but content is not the only thing students need to know in order to be successful test takers as well as productive citizens. This missing link is critical thinking. Students are not only asked to regurgitate facts and processes on standardized tests, they need to be able to coherently describe their thinking. Students need to be able to examine questions logically in order to fully understand what is being asked of them, and in return answer in a comprehensive way (Harlin 2013). This is a skill that students need to be taught, which leads to the question how exactly as teachers can you foster logical thinkers?

A great way to build logical thinkers is through logic puzzles. The wonderful thing about logic puzzles is they can be adapted to any grade level. The next problem occurs when teachers try to decide where to put these puzzles into their content packed schedule. A few suggestions would be as a warm-up, center, or even homework activity. Logical puzzles can also be used to build collaboration skills when assigned to a group to solve. Another great thing about these puzzles is they do not need to be linked to your current content, which allows them to be used as early finisher activities or just as tasks to keep students engaged in the learning.

Along with many benefits of using logic puzzles there are a few mind fields that educators can encounter when using them. Teachers who have ELLs in their classrooms might notice that the language barrier will hinder the student’s ability to complete the assignment. This is due to the fact that logic problems require a command over the English language that these students may not possess. Another problem teacher may encounter is the amount of time that more sophisticated problems will require. A way to combat both of these problems is allowing students to work in groups (Bouchard n.d.).

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