Masthead Logo

Transformations

Volume 5
Issue 1 *Winter 2019*

Article 4

2019

Using Number Properties to inspire teaching and learning in the K-12 Classrooms

Hui Fang H. Su
Nova Southeastern University, shuifang@nova.edu

Farrah Bell
Nova Southeastern University

Denise Gates
Nova Southeastern University

Janice Haramis
Nova Southeastern University

Kristin Hjerpe
Nova Southeastern University

See next page for additional authors

Using Number Properties to inspire teaching and learning in the K-12 Classrooms

Authors
Hui Fang H. Su, Farrah Bell, Denise Gates, Janice Haramis, Kristin Hjerpe, Claude Manigat, and Lourivaldo Da Silva

Using Number Properties to inspire teaching and learning in the K-12 Classroom

Abstract

Where would today's world be without the number system? Indeed, civilization would not have come to as far as in modern times. In fact, the software this paper is being written on is run by a computer that is embedded in numerical codes and commands– something that would not be possible if it were not for the advancement of technology using the modern number system. In this paper, we study the historical development of the modern-day number system. We investigate the classification of numbers and number properties for mathematical use; including standard functions, rules, and examples. As a culmination of our research, we provide a comprehensive activity for classroom use, accompanied by worksheets and a classroom-ready presentation.

Historical Development of Number

The discovery of "number" is unlikely to have been of any one individual or single ethnic group, but rather a plodding awareness in man's cultural advancement about 300,000 years ago. The historical development of number was a long and gradual progression influenced by many cultures, including the Greeks. Evidence shows that our early ancestors began by only counting to two and that anything beyond was designated as "many". This method is still used today, as most people count objects by an arrangement of twos. Unfortunately, due to poor preservation, few records remain but demonstrate that prehistoric man would sometimes record a number by scratching notches with a stick or a piece of animal bone. Such as, in Moravia, a wolf bone dated back to over 30,000 years old, was found with fifty-five cuts deeply incised. Two additional significant artifacts were found in Africa: a 35,000-year-old, baboon fibula with twenty-nine cuts and an Ishango bone with multiplicative entries, dating back to over 30,000 years ago. As technology has evolved and new research emerges, examiners are discovering that the idea of number is far older than we formerly recognized (Boyer, 1968).

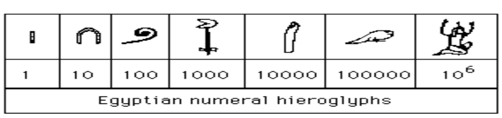
The Ishango bone, baboon fibula, ~30,000 years old, with notches, believed to represent early evidence of counting.

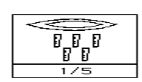
Aristotle wrote: "Or is it because we were born with ten fingers and so because they possess the equivalent of pebbles to the number of their fingers, come to use this number for counting everything else as well?" Yes, indeed, Artistole, counting developed from our nature of five fingers on each hand and foot. The historical development of number is dependent on several contributions from diverse cultures. The French used a base 20 number system and words such as quatre-vingt (four twenties), for the number eighty. The Old Khmer language used five as a point to secure, or "anchor" numbers. For example, after four comes five, five and one, five and two, and so on until ten, beginning at the next "anchor" number. As you can see, the Early European numbers, or more commonly known as Roman Numerals, follow this same trend with anchored numbers by the number five: IV, V, VI, VII, VIII...etc. Evidence from the decimal numbers early on in Ancient India shows us they used the number ten as their anchor. The largest known primary number, sixty, was found in the Babylonia sexagesimal system. The Babylonians cultivated their number system based on the number sixty rather than ten. Surprisingly, traces of their earliest system are alive today with sixty seconds in a minute/hour, and a circle is (6 X 60) (Aczel, 2015).

Number Systems

Numbers are classified into sets, called number systems, and there are five important number systems to consider: natural numbers, integers, rational numbers, real numbers, and complex numbers. The creation of the number system was a leading principal in the progression of our decadic system and many others. Although highly developed, the Mayan number system, one that uses basic twenty group, has an indiscretion with the second order: it's not 20 X 20 = 400 as expected, but 20 X 18 = 360. Theorists suggest that this is a correlation with the division of the Mayan year into eighteen months with each having twenty days, with five extra days. "Number Theory and It's History" illustrates the higher groups in the system as 360 X 20, 360 X 20^2 … etc. Many cultures did not use enormous numbers; in fact, many languages did not go beyond thousands or even hundreds. For example, the Greeks stopped at a myriad (10,000) and Romans, for an extensive period, did not go over 100,000. On the other hand, the Hindus were keen on large numbers, which ultimately resulted in the higher decadic groups to extremely high powers of ten (Ore, 1988).

Around 1800 BC, Egyptians used fractions within their number system with a base of ten. Their ancient Egyptian writing system contained illustrations, hieroglyphs, and they even used these pictures to represent numbers.



This is how they wrote 1/5: 

Ancient Babylonians came up with a sensible representation of fractions as well (around the base of ten), and did it well before the Romans method (Pumfrey, 2011).

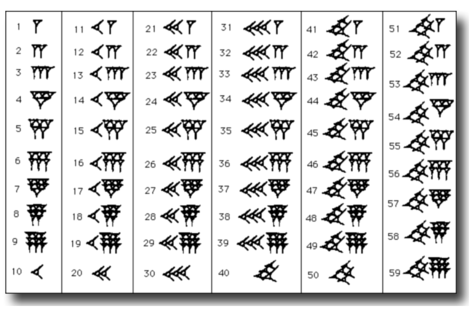
Negative numbers turned up in India around 620 CE, in the works of Brahmagupta (598-670). Brahmagupta symbolized fortunes and debts as positives and negatives. Notably, during this time, the place value system was already in effect in India and zero was used in the Indian Number System. Furthermore, to note the importance of the Greeks never really concentrated on negative numbers since they had a more geometrical approach. Greeks were more centered on numbers that had to be positive (lengths, areas, and volumes, all of which had to be positive) for their proofs of logical agreements (Rogers, 2011).

"A Brief History of Numbers" author Leo Corry, mentions that it was not until the mid-19th century that complex numbers were fully understood by mathematicians. Nonetheless, once complex numbers and their significance was discovered, they were incorporated in mathematics, and other areas, such as physics and engineering, for their countless uses (Corry, 2015). The evolution of complex numbers took almost three hundred years. In 1545, the book *Ars Magna* (The Great Art) by Jerome Cardan, an Italian mathematician, physician, and philosopher, was published. Cardan discusses various rules of algebra and other algebraic procedures for solving cubic and quartic equations. However, when using the cubic formula, or "The Cardan Formula," to solve the example, Cardan stated that the general formula was not applicable in this case (because of the root of -121). Nevertheless, it took Rafael Bombelli (1526-1572), a hydraulic engineer, approximately thirty years after Cardan's work was published, to figure out this problem. Bombelli justified Cardan's formula by introducing complex numbers, thus laying the groundwork. Bombelli's work demonstrated that sometimes the square roots of a negative number could be used to find real solutions. While Bombelli thought complex numbers were worthless, he unknowingly significantly influenced others with his work. In 1620, for instance, Albert Girard claimed an equation may have as many roots as its degree and shortly after, Rene' Descartes contributed the term "imaginary" for these numbers (Complex Numbers).

As outlined, it is evident that numbers and number systems have evolved over time. What follows is a thorough look at some of the number systems discussed.

The Babylonians used two cuneiform symbols and arranged them into fifty-nine base units using a base sixty number system. They used a positional number system like we have today; they organized their numbers into columns. As we know, the first column was the unit column, and it contained any of the fifty-nine base units. The next column contained multiples of sixty, for each of the fifty-nine base units. The third column was used to represent sixty squared or three thousand six hundred; each of the fifty-nine base

units could be placed in the third column (O'Connor & Robertson, 2000).



(*mathisgoodfor you.com)*

The Greeks used a number system that was based on the letters of their alphabet. The Greek alphabet consisted of twenty-four letters and three obsolete letters. Each of the letters was assigned a value from one to nine hundred. To distinct, the numbers between the letters, a special symbol called a keraia was used. The keraia was also used to make larger numbers; it was placed to the lower left of a letter to indicate that the value of the letter should be multiplied by one thousand. Furthermore, a myriad symbol (M) was used to indicate multiples of ten thousand, so even larger numbers could be created. Overall, the Greek number system was a base ten additive system; it added the numeric value of the letters to get a total (O'Connor & Robertson, 2000).

1	α	alpha	10	ι	iota	100	ρ	rho
2	β	beta	20	κ	kappa	200	σ	sigma
3	γ	gamma	30	λ	lambda	300	τ	tau
4	δ	delta	40	μ	mu	400	υ	upsilon
5	ϵ	epsilon	50	ν	nu	500	ϕ	phi
6	ϛ	vau*	60	ξ	xi	600	χ	chi
7	ζ	zeta	70	ο	omicron	700	ψ	psi
8	η	eta	80	π	pi	800	ω	omega
9	θ	theta	90	ϙ	koppa*	900	ϡ	sampi

*vau, koppa, and sampi are obsolete characters

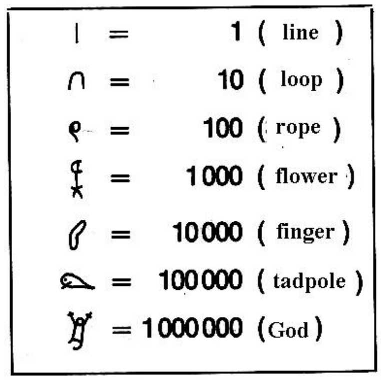
(www.simple-talk.com)

The Romans used seven letters from the Latin alphabet to represent the numbers one, five, ten, fifty, one hundred, five hundred, and one thousand. The Romans placed a line above a letter to multiply its value by one thousand (UNRV History, 2016).

I	1	XXI	21	XLI	41	LXI	61	LXXXI	81
II	2	XXII	22	XLII	42	LXII	62	LXXXII	82
III	3	XXIII	23	XLIII	43	LXIII	63	LXXXIII	83
IV	4	XXIV	24	XLIV	44	LXIV	64	LXXXIV	84
V	5	XXV	25	XLV	45	LXV	65	LXXXV	85
VI	6	XXVI	26	XLVI	46	LXVI	66	LXXXVI	86
VII	7	XXVII	27	XLVII	47	LXVII	67	LXXXVII	87
VIII	8	XXVIII	28	XLVIII	48	LXVIII	68	LXXXVIII	88
IX	9	XXIX	29	XLIX	49	LXIX	69	LXXXIX	89
X	10	XXX	30	L	50	LXX	70	XC	90
XI	11	XXXI	31	LI	51	LXXI	71	XCI	91
XII	12	XXXII	32	LII	52	LXXII	72	XCII	92
XIII	13	XXXIII	33	LIII	53	LXXIII	73	XCIII	93
XIV	14	XXXIV	34	LIV	54	LXXIV	74	XCIV	94
XV	15	XXXV	35	LV	55	LXXV	75	XCV	95
XVI	16	XXXVI	36	LVI	56	LXXVI	76	XCVI	96
XVII	17	XXXVII	37	LVII	57	LXXVII	77	XCVII	97
XVIII	18	XXXVIII	38	LVIII	58	LXXVIII	78	XCVIII	98
XIX	19	XXXIX	39	LIX	59	LXXIX	79	XCIX	99
XX	20	XL	40	LX	60	LXXX	80	C	100
								D	500
								M	1000

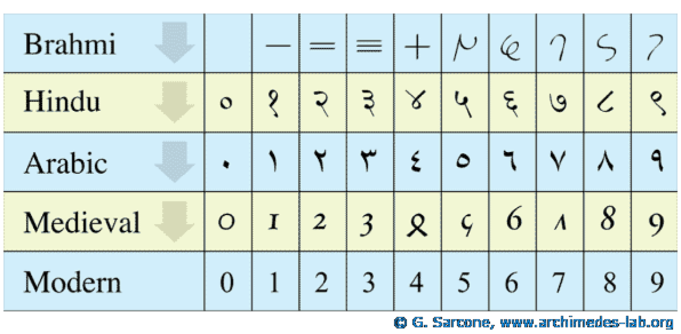
(kreannasandoval.wordpress.com)

The Egyptians used a base ten number system. It was an additive system in which numeric values were created by combining symbols. The symbols for one through nine contained single lines, or strokes of equal number for each symbol and the symbols for ten, one hundred, one thousand, ten thousand, one hundred thousand, and one million were made from objects from their everyday lives. They wrote their numeric symbols from left to right or from right to left; they would also write their numbers vertically in columns (Holt, 2016).



(mpec.sc.mahidol.ac.th)

The Hindu-Arabic is a numeration system much like the system that we use today. The Hindu-Arabic number system uses ten digits that can be utilized in any combination to represent any value. The digits in the number system are zero, one, two, three, four, five, six, seven, eight, and nine. The Hindu-Arabic number system groups by tens – whereas ten ones are replaced by a ten, ten tens are replaced by a hundred, ten hundreds are replaced by a thousand, ten one thousands are replaced by ten thousands, and so on. Also, this number system uses place value, from right to left: ones, tens, hundreds, thousands, and so on (O'Connor & Robertson, 2000).



(www.archimedes-lab.org)

Properties of Numbers

The properties of numbers are the basic rules of our number system. Understanding the properties of numbers is essential to one's ability to solve mathematical problems. First and foremost, let's look at the Properties of Integers. Integers are whole numbers and their opposites. The opposite of a whole number is the negative of the said whole number. The number 0 is also considered an integer, but 0 is the opposite of itself. There are five properties related to integers. The Commutative Property of Addition states that you can add numbers in any order. For example, adding negative two plus three is the same as adding three plus negative two. Therefore, a +b = b + a. The Commutative Property of Multiplication states that you can be able to multiply numbers in any order, without changing the result – the product. For example, four times negative five is the same as negative five times four. Therefore, ab=ba. The Associative Property of Addition states that numbers in a sum can be grouped in any way, with the resulting sum remaining the same. For example, the sum of three and four plus two is the same as the sum of four and two plus three. Therefore, (a +b) + c = (b + c) +a. The Associative Property of Multiplication states that you can group factors in any way and still get the same product. For example, you can multiply negative six and positive two and then multiply that product by two or you can multiply two and positive two and multiply that product by negative six and get the same result. Therefore, (ab)c = a (bc). The Distributive Property applies to a mathematical expression involving addition that is then multiplied by something. It states that you can add first and then multiply or multiply and then add; either way, the multiplication is distributed over all of the terms in the mathematical expression. For example, in the mathematical expression -5(4+2) = (-5 x 5) + (-5 x 2) you can either add the numbers within the parentheses first. Which is four plus two and then multiply the result by negative five or you can multiply negative five and each term of the expression separately and then add the two products together. Therefore, a (b + c) = ab + ac (Math.com, 2005).

In addition to Properties of Integers, there are also Properties of Rational Numbers. Rational numbers are real numbers that can be written as a fraction in the form of a/b, as a ratio. Rational numbers are associate and commutative and under addition and multiplication. The first property of rational numbers is that of the Closure Law. Rational numbers are considered closed under addition, subtraction, and multiplication. If a and b, in a ratio are rational numbers, then the sum, difference and product of the rational numbers are also a rational number. If the rational numbers of a particular ratio satisfy the conditions above, then they satisfy the Closure Law.

CLOSURE LAW

	Addition	Subtraction	Multiplication	Division
Whole Number	X		x	
Integers	X	X	x	
Rational Numbers	X	X	x	

Rational numbers are also commutative under addition and multiplication. If a and b in a ratio are rational numbers, then the Commutative Law under addition applies: $a + b = b + a$ and the Commutative Law under multiplication applies: $a \times b = b \times a$.

COMMUTATIVE LAW

	Addition	Subtraction	Multiplication	Division
Whole Numbers	X		X	
Integers	X		X	
Rational Numbers	X		x	

Rational numbers can also be associative under addition and multiplication. If a, b, and c are rational numbers, then the Associative Law under addition applies: $a + (b+c) = (a +b) + c$ and the Associative Law under multiplication: $a (bc) = (ab) c$.

ASSOCIATIVE LAW

	Addition	Subtraction	Multiplication	Division
Whole Numbers	X		X	
Integers	X		X	
Rational Numbers	X		x	

Furthermore, it is imperative to remember the following additional properties of rational numbers: one is the multiplicative identity for rational numbers, zero is the additive identity for rational numbers, the additive inverse of a rational number pq is –pq and the additive inverse of –pq is pq, and if pq times ab is equal to one, then ab is the reciprocal of pq (Pearson, 2014).

Not only do integers and real numbers have properties that apply to their use, but so do real numbers. Real numbers include whole numbers, rational numbers, or irrational numbers, and they can be positive, negative, or zero. The only types of numbers that are not considered real numbers are imaginary numbers and infinity. There are five properties of real numbers. The Commutative Property of Addition states that you can add numbers in any order and still get the same sum. For example, 8a + 9 = 9 + 8a. The Commutative Property of Multiplication states in any order, you can multiply and still get the same product. For example, 7 x 6 x 3a = 3a x 7 x 6. The Associative Property of Addition states that you can group numbers in an expression in any way and still get the same sum. For example, (3x + 1x) + 9x = 3x + (9x + 1x). The Associative Property of Multiplication states that you can group factors together in any way and still get the same product. For example, 8 x (2 x 3) = (3 x 8) x 2. The Distributive Property applies to mathematical expressions that involve both addition and multiplication. The Distributive Property states that if a term in an expression is multiplied by terms in parentheses, the multiplication needs to be distributed over all terms inside the parentheses. For example, 4x (7 + y) = 28x + 4xy. The Density Property states that you can find another real number that lies between any two real numbers.

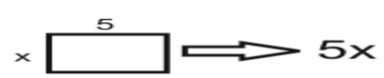
For example, between 7.51 and 7.52, there are the numbers 7.511, 7.512, 7.513, and so on. The Identity Property of Addition states that when zero is added to any number, the sum is the number itself. Zero is referred to as the additive identity. For example, 8y + 0 = 8y. The Identity Property of Multiplication states that the number one multiplied by any other number results in the other number. One is referred to as the multiplicative identity. For example, 8f x 1 = 8f (Brennan, 2002).

Just as there are properties that relate to integers, rationale's, and reals there are also properties that relate to complex numbers. Complex numbers are numbers that are a combination of real numbers and imaginary numbers. Complex numbers are in the form of a + ib, where a is the real numbers and bi are the imaginary number. Imaginary numbers are called imaginary because they lie in the imaginary plane; they arise from taking square roots of negative numbers. The (i) on an imaginary number is equal to the square root of negative one. Imaginary numbers behave like natural numbers when it comes to addition and subtraction. For example, 4i + 6i = 10i and 85i – 5i = 80i. In terms of multiplication, $\sqrt{a} \times \sqrt{a} = a$ Therefore, the following is also true: $\mathrm{i} \times \mathrm{i} = -1$ since **i** = -1, and $\sqrt{-1} \times \sqrt{-1} = -1$. In looking at the division, imaginary numbers can be divided just like any other number if there is only one term: i divided by i equals one or 3i divided by I equals three. If there are two terms divided by two terms, use the complex conjugate: a – bi (Stapel, 2016).

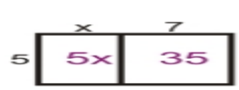
Description of the Activity

In this activity, the twenty-six students will be introduced to mathematical properties that form the foundation of computation. This task is aimed to help students understand the distributive property using area models. Students will build upon their prior knowledge of properties of operations as strategies to multiply and divide rational numbers. According to Lee E. Boyer (1967), clear are the distributive property of multiplication over addition. “The two forms of the distributive property: a (b + c) = ab + ac and (b + c) a = ba + ca. Clearly, if it was worthwhile (and the author thinks it was) to emphasize the two forms of the distributive property in the separate number systems of the natural numbers, the integers, the rational numbers, and the real numbers - in which both forms of the distributive property were always equivalent."

The first part of the Area Representative of the Distributive Property activity can be used to help student recall information regarding the area as a precursor for the distributive property. The first section introduces students to the idea of writing the area of a rectangle as an expression of length x width, even when a variable may represent one more dimension. For example:



According to Arlene Roberts and Jeffrey Chaffee (2010), Distributing and Factoring Using Area it is believed that "students usually grasp the concept of the expanding component rather quickly, but they struggle with the factoring component." The students will show how to model area tiles by multiplying the length and width of a figure together. Also, they explain not only how to complete the step-by-step method but also the reasoning behind it. This sector allows students to begin to piece together some of the fundamental concepts of the distributive property. The teacher suggests that students write the area of each of the figures within the corresponding boxes. For example:



In this key section of the activity, students represent the area of the individual rectangle in two forms to distribute the common factor among all parts of the expression in parentheses. Students love the opportunity to model different ways to solve problems. They also appreciate using the tile manipulatives to create an area model to draw the diagrams in their interactive notebook. The teacher can assign partners or allow students to select their partners. "It is difficult for students to learn to consider, evaluate, and build on the thinking of others, especially when their peers are still developing their own mathematical understandings" (NCTM 2000, p. 63).

It leads to the fundamental concepts of the distributive property. The area as a product section requires students to think about how to represent the area of the entire rectangle without using the area of each of the individual boxes. Area of a rectangle can be created by multiplying the length and width of a figure. The area as a sum division compels students to think about how to symbolize the area of the rectangle while using taking the sum of the areas to find the area of the whole rectangle. For instance, the students would be able to know that the area found by multiplying $(\boldsymbol{x} + \mathbf{7})\ \mathbf{5} = (\boldsymbol{x} + \mathbf{7})$ by the commutative property. The area as a sum of the first rectangle can be found by multiplying the length, x, and the width, 5, together. Thus, the area of the first rectangle is x (5) = 5x by the commutative property. The area of the second rectangle can be found by multiplying the length, 7, by the width, 5. Thus, the area of the second rectangle is 7(5) = 35. To find the total combined area, students must add together the areas of both figures. Therefore, the total combined area is identified as the expression 5x+35. It is helpful to have students to write in their interactive notebook; in fact, it would allow the teacher to determine the different levels student demonstrate understanding.

During the activity, the teacher is monitoring how students are pair-think-sharing responses from the problems and how they are justifying their reasoning. According to Ronald V. McDougall (1967), “the distributive property is one of the most effective instruments we have for achieving these worthwhile objectives”. To accommodate struggling students, the teacher will pose leading questions to help guide them with the lesson. Another accommodation for struggling students, the teacher will have extended the time to finish the task. Students are on different levels in the classroom; for this reason, “some of the students may be ready to move to a visual representation” using distributive property (Morelli 1992). Different demonstrations of the distributive property will reach all learners. According to Lynn Morelli (1992), “several more exercises can be conducted with the students moving between the verbal steps, the numerical example, the visual representation, and the algebraic symbols”.

During the final activity, the teachers will help struggling students comprehend both expressions that they produced from the created questions are equivalent and symbolize the same information in different ways. According to Scott Beckett (1990), “The following activity helps take the mystery out of the distributive property for middle school students. It allows them to build a physical model of the property that they label with its symbolic name”. Students who are working above level will be given more challenging problems to work on with the teacher as I walk around the room. English learner will have more area model to work on to determine lesson comprehension.

Addressing Different Learning Styles

There is strong evidence that proves that working with number tracks or lines helps students develop a better number sense, and being fluent in flexible counting strategies can lay the foundation for effective calculation strategies. Furthermore, recognition of real number systems and fractions, decimals, measuring, and division are all interrelated concepts and ideas (Askew, 2015). To address students’ multiple intelligent and incorporate appropriate different learning styles, teaching real number systems will focus on addressing various avenues. Teaching will focus on the auditory presentation of material for the auditory learners, while assessments in the form of tests, quizzes, and written homework, will address the verbal learners/thinkers.

*See Attached Real Number System Worksheet Handout:

<u>The Real Number System</u>

I. Mark an X for each category that the number applies

II. Construct a number system on construction paper with the numbers below, in order, from least to greatest. Be sure to label your number line.

	Number	Real	Rational	Irrational	Integer	Whole	Natural
1	-5						
2	61%						
3	0						
4	$\pi/3$						
5	2.9						
6	2/9						
7	$\sqrt{6}$						
8	$\sqrt{64}$						
9	1						
10	¼						
11	-2						
12	4.79						
13	$3\pi/4$						
14	99%						
15	5/4						
16	6 ½						
17	13.5						
18	$7\sqrt{2}$						
19	$2\sqrt{3}/3$						
20	2/5						
21	13 5/8						
22	2,000,000						
23	-6982						
24	18.1						
25	-18.1						

Moreover, when teaching a lesson on the real number system, as an educator, it is imperative to ask several types of questions:

- A Comprehension Question—"What is this question asking me?"

  Look for a detailed rationalization that connects these number classifications to the real number system. Do the students understand the differences between real, rational, irrational, integer, whole and natural numbers?

- A Kinesthetic Question—Ask three students to model the first five number classifications, physically on paper. Stand up, demonstrate and explain to me what it looks like.

- A Visual Question—Ask three students to draw a picture of what the next five numbers look like. Draw the numbers, in order, on the board.

As an educator, for students to successfully master mathematical skills and concepts, covering all the various learning styles is vital. One can address different learning styles by incorporating an assortment of questions into the daily curriculum. As well, asking diverse questions achieves a few goals. First, it allows students to articulate a variety of learning methods. Secondly, integrating questions covers a broad range of various levels of thinking skills, such as critical thinking and analytical thinking (Buher).

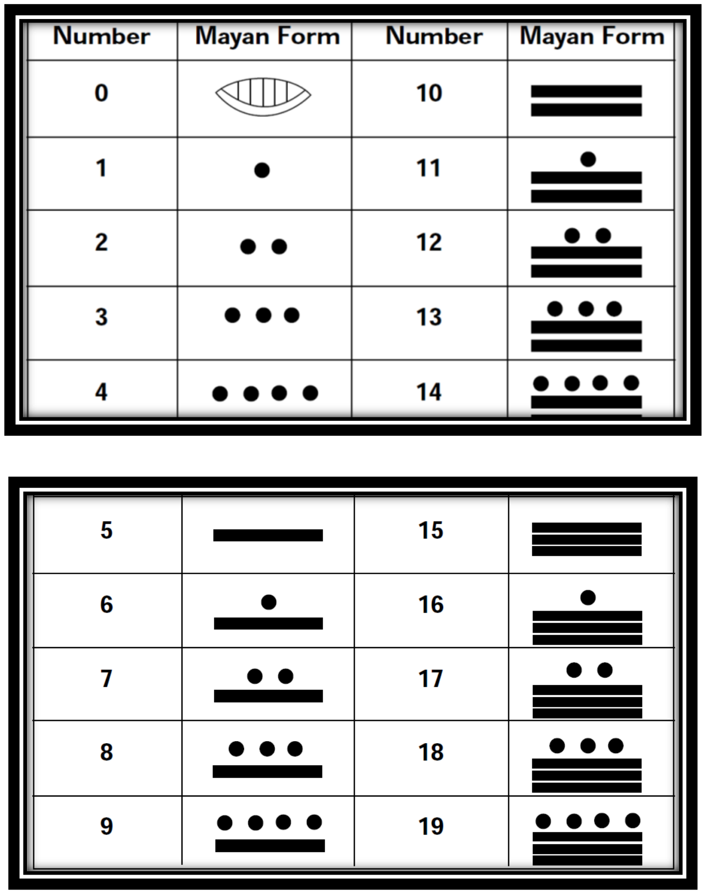
Classroom Use

According to the writer Julie Murgel, the following Mayan Numbers activity help students develop number sense, understand and use the correct vocabulary, and analyze the relationship between numbers and problem-solving situations. Moreover, students link concepts and procedures as they develop and use computational techniques, including mental arithmetic, estimation, paper-and-pencil, calculators, computers, and other manipulatives in problem-solving situations and communicate the reasoning used in solving these problems.

Also, to accommodate the needs of all students, the activity stimulates students to construct and to interpret number meaning through real-world experiences and the use of hands-on materials and relate these meanings to mathematical symbols and numbers. Students model, explain and use the four basic operations—addition, subtraction, multiplication, and division—in problem-solving and real-world situations. During the in-class activity, students are required to identify Mayan numbers, convert a base 10 number to a base 20 number (Mayan) and vice-versa, and use Mayan numbers to add, subtract, multiply, and divide.

The teacher will lecture for few minutes using PowerPoint to facilitate the understanding of visual learners that the Maya used a base 20 number system. They symbolized their numbers using dot and bars; where a dot equaled 1 and a bar equaled 5. After the teacher describes and illustrates the Mayan numeric system, students will practice identifying Mayan numbers from the problems on the board. Then, they will observe how to convert a Mayan number to our number system and vice-versa. After that, students will work in groups of two to complete section 1 and two from worksheet 1.

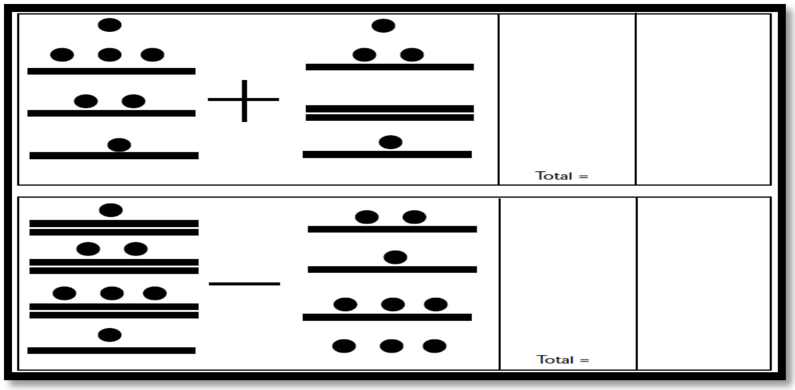
Mayan Number Chart

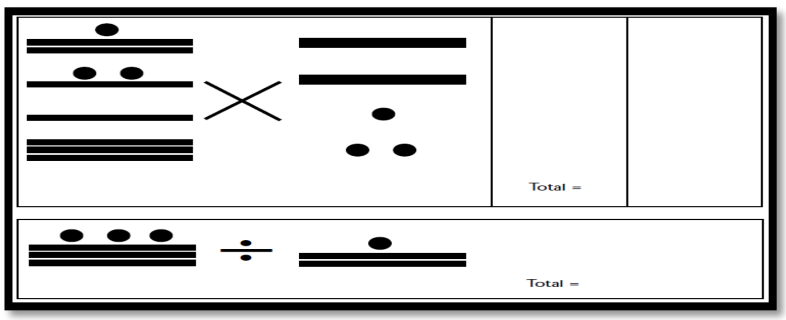


Mayan Numbers: Worksheet 1

Directions: By means of the Mayan numerical system, compute each problem. Be sure the final answer is in Mayan mathematical symbols.

Name: ____________________________________Date_____________________





(Murgel, J., 2000).

Numerical System

According to the article “Number Systems: Naturals, Integers, Rationals, Irrationals, Reals, and Beyond,” the numerical system is categorized as natural numbers, whole numbers, integers, rational numbers, irrational numbers, real numbers, and complex numbers. Some of them have common elements, and some have no intersection at all such as whole and irrational numbers. Also, transcendental numbers, algebraic numbers, and quaternion numbers are considered part of the numerical system (Number Systems: Naturals, Integers, Rationals, Irrationals, Reals, and Beyond, 2014)

The Natural Numbers

The Natural Numbers is known as counting numbers such as 1, 2,3,4,5,6,7,8, etc. There are infinitely many natural numbers. The Whole Numbers differs from the Natural ones because is contain the zero elements. Also, the result of adding two natural numbers will as always be another natural number. For instance, 204 + 200 = 404, and by multiplying two natural numbers, the outcome will always be another natural number such as 200 x 3 = 600. On the other hand, for the division and subtraction cases the statement is not true (Number Systems: Naturals, Integers, Rationals, Irrationals, Reals, and Beyond, 2014).

The Integers

The integers are known as a set of infinite positive and negative numbers including the number zero like {…,-3,-2,-1, 0, 1, 2, 3…}. The integers number is represented by the capital letter Z. Also, whenever two integers are added, subtracted, or multiplied the answers will always be another integer number; however, this statement is no true for the division case such as 9 divides by 2 is equal to 2.5 which is a decimal number (Number Systems: Naturals, Integers, Rationals, Irrationals, Reals, and Beyond, 2014).

The Rational Numbers

The rational number can be defined as every number that is written in the form of a divided b with b different of zero. All elements of integers and natural numbers can be called as rational numbers because the denominator of an integer number are 1. Also, terminate decimals are rational numbers also like 3.245 = 3245/1000 because it can be expressed as a fraction. Repeating decimals can be converted to fraction also such as 0.444 = 4/9. To conclude, "The set of rational numbers is closed under all four basic operations, that is, given any two rational numbers, their sum, difference, product, and quotient is also a rational number (as long as we don't divide by 0)," (Number Systems: Naturals, Integers, Rationals, Irrationals, Reals, and Beyond, 2014).

The Irrational Numbers

On the other hand, the irrational number cannot be expressed in the form of a divided by b with b different of zero. The pattern of an irrational number can be identified as a decimal number that it never ends or repeats. Going back to the Ancient history, the Greeks made great discoveries regards the irrational numbers. They also proved that not all numbers are rational and that certain equations cannot be solved by ratios of integers. For instance, solving the equation $x^2 = 2$ the solution will be x = 1.41422135624 etc. By squaring the solution a number close to 2 can be found, but it will never hit exactly the number 2. So, the conclusion is that square root of 2 is an irrational number that is equal to decimal numbers that repeat without a pattern (Number Systems: Naturals, Integers, Rationals, Irrationals, Reals, and Beyond, 2014).

Also, "other famous irrational numbers are **the golden ratio**, a number with great importance to biology: $\frac{1+\sqrt{5}}{2} = 1.61803398874989$ *And,* π (pi), the ratio of the circumference of a circle to its diameter: $\pi = 3.14159265358979$. And e, the most important number in calculus: $e = 2.71828182845904...$ and e is the most important number in calculus," (Number Systems: Naturals, Integers, Rationals, Irrationals, Reals, and Beyond, 2014).

The Real Numbers

The real numbers are known as a set of natural numbers, integers, rational and irrational numbers. As we know, the number line contains all real numbers, and there are infinitely many real numbers. Also, "The "smaller", or a **countable** infinity of the integers and rationals is sometimes called $\aleph_0$ (alef-naught), and the **uncountable** infinity of the reals is called $\aleph_1$ (alef-one). There are even "bigger" infinities, but you should take a set theory class for that," (Number Systems: Naturals, Integers, Rationals, Irrationals, Reals, and Beyond, 2014).

The Complex Numbers

Also known as the imaginary numbers, the complex numbers are expressed as a set of a + b*i* which a and b stands for rational numbers. Also, *i* is the imaginary unit such as the square root of -1 is only possible to extract by substituting the negative sign by *i* square. The complex numbers can be written as a capital letter C, and it has a critical function regarding solving any polynomial p(x) with real number coefficients (Number Systems: Naturals, Integers, Rationals, Irrationals, Reals, and Beyond, 2014)

Conclusion

After examining the historical development of numbers and various number systems, it is evident that the use of numbers themselves and the individual number systems have significantly evolved over time. The evolution of the vast array of number systems has provided humanity with the opportunity to employ the use of the number systems and their corresponding properties. Furthermore, it is essential that students develop an understanding of the properties associated with the five important sets of numbers in our number system: natural numbers, integers, rational numbers, real numbers, and complex numbers. Students can develop an understanding of the five essential sets of numbers and their corresponding properties through the use of various learning strategies; activities that allow for individual and cooperative learning groups. Understanding the properties of different sets of numbers lays the foundation for higher order thinking in mathematics: critical thinking skills, analyzing and problem-solving skills, and application to everyday situations.

References

1. Aczel, A. D. (2015). *Finding Zero: A Mathematician's Odyssey to Uncover the Origins of Numbers*. St. Martin's Press.
2. Askew, M. (2015). *A Practical Guide to Transforming Primary Mathematics: Activities and Tasks That Really Work*. Retrieved February 3, 2016.
3. Beckett, S. (1990). Distributive property. *The Mathematics Teacher*, *83*(5), 385–399. Retrieved from http://www.jstor.org/stable/27966715
4. Boyer, C. B. (1968). *A History of Mathematics*. New York: Wiley.
5. Boyer, L. E. (1967). The distributive property. *The Arithmetic Teacher*, *14*(7), 566–569. Retrieved from http://www.jstor.org/stable/41185659
6. Brennan, James W. (2002). *Properties of real numbers*. Retrieved from jamesbrennan.org/algebra/numbers/properties_of_real_numbers1.htm
7. Buher, G., & Walbert, D. (n.d.). 3 Assessing the learning process. Retrieved February 03, 2016, from http://www.learnnc.org/lp/editions/mathmultintell/645
8. Complex Numbers. (n.d.). Retrieved February 03, 2016, from http://www.und.edu/instruct/lgeller/complex.html
9. Corry, L. (2015). *A Brief History of Numbers*. Oxford University Press.
10. Holt, Lloyd. (nd). *The egyptian number system*. Retrieved from http://www.math.wichita.edu/history/topics/num-sys.html#egypt
11. Math.com. (2005). *Properties of integers*. Retrieved from www.math.com
12. Math Is Fun. (2013). *Rational numbers*. Retrieved from www.mathisfun.com
13. McDOUGALL, R. V. (1967). Don't sell short the distributive property. *The Arithmetic Teacher*, *14*(7), 570–572. Retrieved from http://www.jstor.org/stable/41185661

14. Morelli, L. (1992). A Visual Approach to Algebra Concepts. *The Mathematics Teacher*, *85*(6), 434–437. Retrieved from http://www.jstor.org/stable/27967693

15. Murgel, J. (2000). Mayan mathematics and architecture. *Goals 2000 Partnership for Educating Colorado Students.* Retrieved from http://www.dpsk12.org/programs/almaproject/pdf/MayanMathematics.pdf

16. National Council of Teachers of Mathematics (NCTM). Principles and Standards for School Mathematics. Reston, VA: NCTM, 2000

17. Number Systems: Naturals, Integers, Rationals, Irrationals, Reals, and Beyond (2014). In Hotmath online. Retrieved from http://hotmath.com/hotmath_help/topics/number-systems.html.

18. O'Connor, J.J. & Robertson, E.F. (2000). *Babylonian numerals*. Retrieved from http://www-history.mcs.st-andrews.ac.uk/HistTopics/Babylonian_numerals.html

19. Ore, O. (1988). *Number Theory and Its History*. New York: Dover Publications.

20. Pearson. (2014). *Properties of rational numbers*. Retrieved from math.tutorcircle.com/number-senseproperties-of-ratoinal-numbers.html.

21. Pumfrey, L. (2011, February). History of Fractions. Retrieved February 03, 2016, from https://nrich.maths.org/2515

22. Rogers, L. (2011, February). The History of Negative Numbers. Retrieved February 03, 2016, from https://nrich.maths.org/5961

23. Stapel, Elizabeth. (2016). *Complex numbers*. Retrieved from http://www.purplemath.com/modules/complex.htm.

24. Su, H. F., Su, T.C. (2004). From Arithmetic to Algebra: An Interdisciplinary Approach to Teaching Pre K through 8th Grade Mathematics. Houghton Mifflin Publishing Co. College textbook. ISBN: 0-618-52305-7

25. UNRV History. (2016). *Roman numerals*. Retrieved from http://www.unrv.com/culture/roman-numerals.php