

Differentiation of Solutions of Second Order BVPs with Integral Boundary Conditions

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The Boundary Value Problem

Our concern is characterizing derivatives of solutions to the second order nonlocal boundary value problem

$$y'' = f(x, y, y'), \quad a < x < b, \quad (1)$$

satisfying

$$y(x_1) = y_1, \quad y(x_2) + \int_c^d ry(x)dx = y_2, \quad (2)$$

where $a < x_1 < c < d < x_2 < b$, and $y_1, y_2, r \in \mathbb{R}$ with respect to the boundary parameters.

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The Variational Equation

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Definition

Given a solution $y(x)$ of (1), we define the *variational equation along $y(x)$* by

$$z'' = \frac{\partial f}{\partial u_1}(x, y(x), y'(x))z + \frac{\partial f}{\partial u_2}(x, y(x), y'(x))z'. \quad (3)$$

Assumptions on Equation (1)

We require that

(i) $f(x, u_1, u_2) : (a, b) \times \mathbb{R}^2 \rightarrow \mathbb{R}$ is continuous,

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Assumptions on Equation (1)

We require that

- (i) $f(x, u_1, u_2) : (a, b) \times \mathbb{R}^2 \rightarrow \mathbb{R}$ is continuous,
- (ii) for $i = 1, 2$, $\partial f / \partial u_i(x, u_1, u_2) : (a, b) \times \mathbb{R}^2 \rightarrow \mathbb{R}$ is continuous, and

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Assumptions on Equation (1)

We require that

- (i) $f(x, u_1, u_2) : (a, b) \times \mathbb{R}^2 \rightarrow \mathbb{R}$ is continuous,
- (ii) for $i = 1, 2$, $\partial f / \partial u_i(x, u_1, u_2) : (a, b) \times \mathbb{R}^2 \rightarrow \mathbb{R}$ is continuous, and
- (iii) solutions of initial value problems for (1) extend to (a, b) .

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Assumptions on Equation (1)

We require that

- (i) $f(x, u_1, u_2) : (a, b) \times \mathbb{R}^2 \rightarrow \mathbb{R}$ is continuous,
- (ii) for $i = 1, 2$, $\partial f / \partial u_i(x, u_1, u_2) : (a, b) \times \mathbb{R}^2 \rightarrow \mathbb{R}$ is continuous, and
- (iii) solutions of initial value problems for (1) extend to (a, b) .

(iii) is not a necessary condition but lets us avoid continually making statements about maximal intervals of existence inside (a, b) .

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Uniqueness for Equation (1)

- (iv) Given $a < x_1 < c < d < x_2 < b$ and $r \in \mathbb{R}$, if $y(x_1) = z(x_1)$ and $y(x_2) + \int_c^d ry(x)dx = z(x_2) + \int_c^d ry(x)dx$ where $y(x)$ and $z(x)$ are solutions of (1), then, on (a, b) ,

$$y(x) \equiv z(x).$$

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Uniqueness for the Variational Equation

- (v) Given $a < x_1 < c < d < x_2 < b$ and $r \in \mathbb{R}$ and a solution $y(x)$ of (1), if $u(x_1) = 0$ and $u(x_2) + \int_c^d ry(x)dx = 0$, where $u(x)$ is a solution of (3) along $y(x)$, then, on (a, b) ,

$$u(x) \equiv 0.$$

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A Theorem of Peano

Theorem

Assume that, with respect to (1), conditions (i)-(iii) are satisfied. Let $x_0 \in (a, b)$ and $y(x) := y(x, x_0, c_1, c_2)$ denote the solution of (1) satisfying the initial conditions $y(x_0) = c_1, y'(x_0) = c_2$.

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A Theorem of Peano

Theorem

Assume that, with respect to (1), conditions (i)-(iii) are satisfied. Let $x_0 \in (a, b)$ and $y(x) := y(x, x_0, c_1, c_2)$ denote the solution of (1) satisfying the initial conditions $y(x_0) = c_1$, $y'(x_0) = c_2$. Then,

(a) for $i = 1, 2$, $\frac{\partial y}{\partial c_i}(x)$ exists on (a, b) , and

$\alpha_i(x) := \frac{\partial y}{\partial c_i}(x)$ is the solution of the variational equation (3) along $y(x)$ satisfying the respective initial conditions

$$\alpha_1(x_0) = 1, \quad \alpha_1'(x_0) = 0,$$

$$\alpha_2(x_0) = 0, \quad \alpha_2'(x_0) = 1.$$

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(b) $\frac{\partial y}{\partial x_0}(x)$ exists on (a, b) , and $\beta(x) := \frac{\partial y}{\partial x_0}(x)$ is the solution of the variational equation (3) along $y(x)$ satisfying the initial conditions

$$\beta(x_0) = -y'(x_0), \quad \beta'(x_0) = -y''(x_0).$$

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(b) $\frac{\partial y}{\partial x_0}(x)$ exists on (a, b) , and $\beta(x) := \frac{\partial y}{\partial x_0}(x)$ is the solution of the variational equation (3) along $y(x)$ satisfying the initial conditions

$$\beta(x_0) = -y'(x_0), \quad \beta'(x_0) = -y''(x_0).$$

(c) $\frac{\partial y}{\partial x_0}(x) = -y'(x_0) \frac{\partial y}{\partial c_1}(x) - y''(x_0) \frac{\partial y}{\partial c_2}(x).$

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Continuous Dependence for BVPs

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Theorem

Assume (i)-(iv) are satisfied with respect to (1). Let $y(x)$ be a solution of (1) on (a, b) , and let

$a < \alpha < x_1 < c < d < x_2 < \beta < b$ and $y_1, y_2, r \in \mathbb{R}$ be given. Then, there exists a $\delta > 0$ such that, for

$i = 1, 2$, $|x_i - t_i| < \delta$, $|c - \xi| < \delta$, $|d - \Delta| < \delta$, $|r - \rho| < \delta$, $|u(x_1) - y_1| < \delta$, and $|u(x_2) + \int_c^d ru(x)dx - y_2| < \delta$, there exists a unique solution $u_\delta(x)$ of (1) such that $u_\delta(t_1) = y_1$ and $u_\delta(t_2) + \int_\xi^\Delta \rho u_\delta(x)dx = y_2$ and, for $i = 1, 2$, $\{u_\delta^{(i)}(x)\}$ converges uniformly to $u^{(i)}(x)$ as $\delta \rightarrow 0$ on $[\alpha, \beta]$.

Theorem

Assume (i)-(iv) are satisfied with respect to (1). Let $y(x)$ be a solution of (1) on (a, b) , and let $a < \alpha < x_1 < c < d < x_2 < \beta < b$ and $y_1, y_2, r \in \mathbb{R}$ be given. Then, there exists a $\delta > 0$ such that, for $i = 1, 2$, $|x_i - t_i| < \delta$, $|c - \xi| < \delta$, $|d - \Delta| < \delta$, $|r - \rho| < \delta$, $|u(x_1) - y_1| < \delta$, and $|u(x_2) + \int_c^d ru(x)dx - y_2| < \delta$, there exists a unique solution $u_\delta(x)$ of (1) such that $u_\delta(t_1) = y_1$ and $u_\delta(t_2) + \int_\xi^\Delta \rho u_\delta(x)dx = y_2$ and, for $i = 1, 2$, $\{u_\delta^{(i)}(x)\}$ converges uniformly to $u^{(i)}(x)$ as $\delta \rightarrow 0$ on $[\alpha, \beta]$.

For a typical proof, we refer [5].

Theorem

Assume conditions (i)-(v) are satisfied. Let $y(x)$ be a solution of (1) on (a, b) . Let $a < x_1 < c < d < x_2 < b$ and $y_1, y_2, r \in \mathbb{R}$ be given so that

$$y(x) = y(x, x_1, x_2, y_1, y_2, c, d, r),$$

where

$$y(x_1) = y_1, \quad y(x_2) + \int_c^d ry(x)dx = y_2.$$

Then,

- (a) for $i = 1, 2$, $u_i(x) := \frac{\partial y}{\partial y_i}(x)$ exists on (a, b) and is the solution of the variational equation (3) along $y(x)$ satisfying the respective boundary conditions

$$u_1(x_1) = 1 \text{ and } u_1(x_2) + \int_c^d ru_1(x)dx = 0,$$

$$u_2(x_1) = 0 \text{ and } u_2(x_2) + \int_c^d ru_2(x)dx = 1.$$

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Then,

- (a) for $i = 1, 2$, $u_i(x) := \frac{\partial y}{\partial y_i}(x)$ exists on (a, b) and is the solution of the variational equation (3) along $y(x)$ satisfying the respective boundary conditions

$$u_1(x_1) = 1 \text{ and } u_1(x_2) + \int_c^d ru_1(x)dx = 0,$$

$$u_2(x_1) = 0 \text{ and } u_2(x_2) + \int_c^d ru_2(x)dx = 1.$$

- (b) for $i = 1, 2$, $z_i(x) := \frac{\partial y}{\partial x_i}(x)$ exists on (a, b) and is the solution of the variational equation (3) along $y(x)$ satisfying the respective boundary conditions

$$z_1(x_1) = -y'(x_1) \text{ and } z_1(x_2) + \int_c^d rz_1(x)dx = 0,$$

$$z_2(x_1) = 0 \text{ and } z_2(x_2) + \int_c^d rz_2(x)dx = -y'(x_2).$$

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(c) $C(x) := \frac{\partial y}{\partial c}(x)$ exists on (a, b) and is the solution of the variational equation (3) along $y(x)$ satisfying the boundary conditions

$$C(x_1) = 0 \text{ and } C(x_2) + \int_c^d rC(x)dx = -ry(c).$$

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- (c) $C(x) := \frac{\partial y}{\partial c}(x)$ exists on (a, b) and is the solution of the variational equation (3) along $y(x)$ satisfying the boundary conditions

$$C(x_1) = 0 \text{ and } C(x_2) + \int_c^d rC(x)dx = -ry(c).$$

- (d) $D(x) := \frac{\partial y}{\partial d}(x)$ exists on (a, b) and is the solution of the variational equation (3) along $y(x)$ satisfying the boundary conditions

$$D(x_1) = 0 \text{ and } D(x_2) + \int_c^d rD(x)dx = ry(d).$$

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- (c) $C(x) := \frac{\partial y}{\partial c}(x)$ exists on (a, b) and is the solution of the variational equation (3) along $y(x)$ satisfying the boundary conditions

$$C(x_1) = 0 \text{ and } C(x_2) + \int_c^d rC(x)dx = -ry(c).$$

- (d) $D(x) := \frac{\partial y}{\partial d}(x)$ exists on (a, b) and is the solution of the variational equation (3) along $y(x)$ satisfying the boundary conditions

$$D(x_1) = 0 \text{ and } D(x_2) + \int_c^d rD(x)dx = ry(d).$$

- (e) $R(x) := \frac{\partial y}{\partial r}(x)$ exists on (a, b) and is the solution of the variational equation (3) along $y(x)$ satisfying the boundary conditions

$$R(x_1) = 0 \text{ and } R(x_2) + \int_c^d rR(x)dx = - \int_c^d y(x)dx.$$

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Proof of Part (a)

We only provide the proof of part (a) as the others are proven in much the same way.

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Proof of Part (a)

We only provide the proof of part (a) as the others are proven in much the same way. For part (a), we will provide the argument for $\partial y/\partial y_1$ as $\partial y/\partial y_2$ is quite similar.

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Proof of Part (a)

We only provide the proof of part (a) as the others are proven in much the same way. For part (a), we will provide the argument for $\partial y/\partial y_1$ as $\partial y/\partial y_2$ is quite similar.

Denote $y(x, x_1, x_2, y_1, y_2, c, d, r)$ by $y(x, y_1)$.

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Proof of Part (a)

We only provide the proof of part (a) as the others are proven in much the same way. For part (a), we will provide the argument for $\partial y/\partial y_1$ as $\partial y/\partial y_2$ is quite similar.

Denote $y(x, x_1, x_2, y_1, y_2, c, d, r)$ by $y(x, y_1)$.

Let $\delta > 0$ be as in the Continuous Dependence Theorem, $0 < |h| < \delta$ be given, and define the difference quotient

$$u_{1h}(x) = \frac{1}{h}[y(x, y_1 + h) - y(x, y_1)].$$

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Note that for every $h \neq 0$,

$$u_{1h}(x_1) = \frac{1}{h}[y(x_1, y_1 + h) - y(x_1, y_1)] = \frac{1}{h}[y_1 + h - y_1] = 1$$

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Note that for every $h \neq 0$,

$$u_{1h}(x_1) = \frac{1}{h}[y(x_1, y_1 + h) - y(x_1, y_1)] = \frac{1}{h}[y_1 + h - y_1] = 1$$

and

$$\begin{aligned}u_{1h}(x_2) + \int_c^d ru_{1h}(x) dx &= \frac{1}{h}[y(x_2, y_1 + h) + \int_c^d ry(x, y_1 + h) dx \\ &\quad - y(x_2, y_1) - \int_c^d ry(x, y_1) dx] \\ &= \frac{1}{h}[y_2 - y_2] = 0.\end{aligned}$$

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We want to show that $u_{1h}(x)$ is a solution of the variational equation (3). So, we view $y(x)$ in terms of the solution of an initial value problem at x_1 .

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We want to show that $u_{1h}(x)$ is a solution of the variational equation (3). So, we view $y(x)$ in terms of the solution of an initial value problem at x_1 .

Therefore, let

$$\mu = y'(x_1, y_1)$$

and

$$\nu = \nu(h) = y'(x_1, y_1 + h) - \mu.$$

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We want to show that $u_{1h}(x)$ is a solution of the variational equation (3). So, we view $y(x)$ in terms of the solution of an initial value problem at x_1 .

Therefore, let

$$\mu = y'(x_1, y_1)$$

and

$$\nu = \nu(h) = y'(x_1, y_1 + h) - \mu.$$

Then,

$$y(x) = u(x, x_1, y_1, \mu),$$

and we have

$$u_{1h}(x) = \frac{1}{h} [u(x, x_1, y_1 + h, \mu + \nu) - u(x, x_1, y_1, \mu)].$$

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Next, by utilizing a telescoping sum, we have

$$u_{1h}(x) = \frac{1}{h} [u(x, x_1, y_1 + h, \mu + \nu) - u(x, x_1, y_1, \mu + \nu) \\ + u(x, x_1, y_1, \mu + \nu) - u(x, x_1, y_1, \mu)].$$

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Next, by utilizing a telescoping sum, we have

$$u_{1h}(x) = \frac{1}{h} [u(x, x_1, y_1 + h, \mu + \nu) - u(x, x_1, y_1, \mu + \nu) \\ + u(x, x_1, y_1, \mu + \nu) - u(x, x_1, y_1, \mu)].$$

By the Mean Value Theorem, we obtain

$$u_{1h}(x) = \frac{1}{h} \left[\frac{\partial u}{\partial y_1}(x, u(x, x_1, y_1 + \bar{h}, \mu + \nu))(y_1 + h - y_1) \right. \\ \left. + \frac{\partial u}{\partial \mu}(x, u(x, x_1, y_1, \mu + \bar{\nu}))(\mu + \nu - \mu) \right],$$

where $y_1 + \bar{h}$ is between y_1 and $y_1 + h$, and $\mu + \bar{\nu}$ is between μ and $\mu + \nu$.

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Furthermore, by Peano's Theorem, we have

$$u_{1h}(x) = \frac{1}{h} [\alpha_1(x, u(x, x_1, y_1 + \bar{h}, \mu + \nu)) \cdot h \\ + \alpha_2(x, u(x, x_1, y_1, \mu + \bar{\nu})) \cdot \nu],$$

where for $i = 1, 2$, $\alpha_i(x, y(\cdot))$ is the solution of the variational equation (1) along $u(\cdot)$ satisfying respectively

$$\alpha_1(x_1) = 1, \quad \alpha_1'(x_1) = 0,$$

$$\alpha_2(x_1) = 0, \quad \alpha_2'(x_1) = 1.$$

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Simplifying,

$$u_{1h}(x) = \alpha_1(x, u(x, x_1, y_1 + \bar{h}, \mu + \nu)) + \frac{\nu}{h} \alpha_2(x, u(x, x_1, y_1, \mu + \bar{\nu})).$$

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Simplifying,

$$u_{1h}(x) = \alpha_1(x, u(x, x_1, y_1 + \bar{h}, \mu + \nu)) + \frac{\nu}{h} \alpha_2(x, u(x, x_1, y_1, \mu + \bar{\nu})).$$

Thus, to show $\lim_{h \rightarrow 0} u_{1h}(x)$ exists, it suffices to show $\lim_{h \rightarrow 0} \frac{\nu}{h}$ exists.

By hypothesis (v), that $\alpha_2(x, u(\cdot))$ is a nontrivial solution of (3) along $u(\cdot)$, and $\alpha_2(x_1, u(\cdot)) = 0$, we have

$$\alpha_2(x_2, u(\cdot)) + \int_c^d r\alpha_2(x, u(\cdot))dx \neq 0.$$

Recall,

$$u_{1h}(x_2) + \int_c^d ru_{1h}(x)dx = 0.$$

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By hypothesis (v), that $\alpha_2(x, u(\cdot))$ is a nontrivial solution of (3) along $u(\cdot)$, and $\alpha_2(x_1, u(\cdot)) = 0$, we have

$$\alpha_2(x_2, u(\cdot)) + \int_c^d r \alpha_2(x, u(\cdot)) dx \neq 0.$$

Recall,

$$u_{1h}(x_2) + \int_c^d r u_{1h}(x) dx = 0.$$

Thus,

$$\begin{aligned} & \alpha_1(x_2, u(x, x_1, y_1, \mu)) + \frac{\nu}{h} \alpha_2(x_2, u(x, x_1, y_1, \mu)) \\ & + \int_c^d r [\alpha_1(x, u(x, x_1, y_1, \mu)) + \frac{\nu}{h} \alpha_2(x, u(x, x_1, y_1, \mu))] dx = 0. \end{aligned}$$

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Hence, we obtain

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{\nu}{h} &= \frac{-\alpha_1(x_2, u(x, x_1, y_1, \mu)) - \int_C^d r \alpha_1(x, u(x, x_1, y_1, \mu))}{\alpha_2(x_2, u(x, x_1, y_1, \mu)) + \int_C^d r \alpha_2(x, u(x, x_1, y_1, \mu)) dx} \\ &= \frac{-\alpha_1(x_2, y(\cdot)) - \int_C^d r \alpha_1(x, y(\cdot))}{\alpha_2(x_2, y(\cdot)) + \int_C^d r \alpha_2(x, y(\cdot)) dx} \\ &:= U. \end{aligned}$$

Now let

$$u_1(x) = \lim_{h \rightarrow 0} u_{1h}(x),$$

and note by construction of $u_{1h}(x)$,

$$u_1(x) = \frac{\partial y}{\partial y_1}(x).$$

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Now let

$$u_1(x) = \lim_{h \rightarrow 0} u_{1h}(x),$$

and note by construction of $u_{1h}(x)$,

$$u_1(x) = \frac{\partial y}{\partial y_1}(x).$$

Furthermore,

$$u_1(x) = \lim_{h \rightarrow 0} u_{1h}(x) = \alpha_1(x, y(x)) + U\alpha_2(x, y(x))$$

which is a solution of the variational equation (3) along $y(x)$.

In addition,

$$u_1(x_1) = \lim_{h \rightarrow 0} u_{1h}(x_1) = \lim_{h \rightarrow 0} 1 = 1,$$

and

$$\begin{aligned} u_1(x_2) + \int_c^d ru_1(x) dx &= \lim_{h \rightarrow 0} \left[u_{1h}(x_2) + \int_c^d ru_{1h}(x) dx \right] \\ &= \lim_{h \rightarrow 0} 0 = 0. \end{aligned}$$

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In addition,

$$u_1(x_1) = \lim_{h \rightarrow 0} u_{1h}(x_1) = \lim_{h \rightarrow 0} 1 = 1,$$

and

$$\begin{aligned} u_1(x_2) + \int_c^d ru_1(x) dx &= \lim_{h \rightarrow 0} \left[u_{1h}(x_2) + \int_c^d ru_{1h}(x) dx \right] \\ &= \lim_{h \rightarrow 0} 0 = 0. \end{aligned}$$

This completes the proof for part (a).

Analogue to Part (c) of Peano

Corollary

Under the assumptions of the previous theorem, we have

$$(a) \quad \frac{\partial y}{\partial x_i} = -y'(x_i) \frac{\partial y}{\partial y_i}, \text{ for } i = 1, 2,$$

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Analogue to Part (c) of Peano

Corollary

Under the assumptions of the previous theorem, we have

$$(a) \quad \frac{\partial y}{\partial x_i} = -y'(x_i) \frac{\partial y}{\partial y_i}, \text{ for } i = 1, 2,$$

$$(b) \quad \frac{\partial y}{\partial c} = -\frac{y(c)}{y(d)} \cdot \frac{\partial y}{\partial d}, \text{ and}$$

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Analogue to Part (c) of Peano

Corollary

Under the assumptions of the previous theorem, we have

$$(a) \quad \frac{\partial y}{\partial x_i} = -y'(x_i) \frac{\partial y}{\partial y_i}, \text{ for } i = 1, 2,$$

$$(b) \quad \frac{\partial y}{\partial c} = -\frac{y(c)}{y(d)} \cdot \frac{\partial y}{\partial d}, \text{ and}$$

$$(c) \quad \frac{\partial y}{\partial c} = \frac{ry(c)}{\int_c^d y(x) dx} \cdot \frac{\partial y}{\partial r}$$

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Analogue to Part (c) of Peano

Corollary

Under the assumptions of the previous theorem, we have

$$(a) \quad \frac{\partial y}{\partial x_i} = -y'(x_i) \frac{\partial y}{\partial y_i}, \text{ for } i = 1, 2,$$

$$(b) \quad \frac{\partial y}{\partial c} = -\frac{y(c)}{y(d)} \cdot \frac{\partial y}{\partial d}, \text{ and}$$

$$(c) \quad \frac{\partial y}{\partial c} = \frac{ry(c)}{\int_c^d y(x) dx} \cdot \frac{\partial y}{\partial r}$$

The proof is a result of the dimensionality of the solution space for the variational equation.

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THANK YOU!

Introduction

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Proof

Corollary

References