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The Importance of Using Manipulatives in Teaching Math Today

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Abstract

This paper explores the research and use of mathematics manipulatives in the teaching of mathematics today during an age of technology and standardized testing. It looks at the drawbacks and cautions educators as they use math manipulatives in their instruction. It also explores some cognitive concerns as a teacher goes about teaching with math manipulatives. The paper shares many commonly used math manipulatives used in today's classrooms and matches them up to some of the Common Core Math Standards that are taught today in classrooms in the USA and around the world.

Keywords: Mathematics, Teaching, Manipulatives, Concrete, Standards, Research

Introduction

This article explores the factors that contribute to teacher use of manipulatives in their instructional math lessons. Math manipulatives are physical objects that are designed to represent explicitly and concretely mathematical ideas that are abstract (Moyer, 2001). Math manipulatives have been around for years. The Montessori Schools have long advocated teaching using concrete objects along with Piaget's emphasis on teaching from the concrete, to the representational, to lastly the abstract, in order to help young learners make sense of their mathematics understanding. George Cuisenaire (1891–1975), a Belgian educator, is famed for his development of the Cuisenaire Rods used today to help teach fraction concepts along with other math ideas; these were developed in the 1950's. Later on, many other math didactics came out of these ideas and lead to the Cuisenaire Math Manipulative Company. Today, there are many commercially made math manipulatives that fill the shelves in most school classrooms.

This paper will build upon previous research that investigates how teachers use math manipulatives in their instructional lessons. Moyer (2004) states that some teachers use manipulatives in an effort to reform their teaching of mathematics without reflecting on how the use of representations may change their own mathematics instruction. Baroody (1989) asserts that Piagetian theory does not state that students must operate on something concrete to construct meaning, although it does suggest that they should manipulate something familiar and reflect on these physical or mental actions. The actively engaged thinking is the component imperative to student learning. Ball (1992) posits that manipulative usage is widely accepted as an effective way to teach mathematics, although there is little effort given toward helping teachers ensure their students make the correct connections between the materials and the underlying mathematical concepts.

Guiding Questions about Using Math Manipulatives

These questions give rise to others, such as: Do grade level curricular differences influence the use of manipulatives when teaching mathematics? What are the cognitive consequences of instructional guidance accompanying manipulatives? Is the manipulative used in such a way that it requires reflection or thought on the part of the student? Is the student making correct connections between the manipulative and the knowledge it is meant to convey? And, as raised by Marley and Carbonneau (2014), what is the value added by various instructional factors that may accompany math manipulatives?

Mathematics Standards

Many new state standards, such as the Florida Math Standards, the *Common Core Math Standards* (National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO), 2010), along with the National Council for Teachers of Mathematics (NCTM, 2010) call for the usage of representational models as a significant area of practice in mathematics instruction. Representations can be interpreted in many ways, such as illustrations, virtual manipulatives, and physical hands-on manipulatives or didactics.

Virtual manipulatives are a computer-based rendition of common mathematics manipulatives and tools (Dorward, 2002). They have become quite popular, convenient, and efficient over the past few years and are deserving of a thorough literature review and study on their own, although they are beyond the scope of this study.

Among the many theorists who provide the foundational basis for using math manipulatives in instructional lessons are Piaget (1952), who believed children cannot comprehend abstract math through explanations and lectures only, and that they need experiences with models and instruments in order to grasp the mathematical concepts being taught. Brunner (1960) believed that students' early experiences and interactions with physical objects formed the basis for later learning at the abstract level. This type of hands-on learning is often referred to as constructivism, and is the basis for integrating math manipulatives into instructional math lessons. These foundation researchers provided guidance for the common use of math manipulatives in many math classrooms today.

Research on Math Manipulatives

Deborah Ball (1992) references a story from her own teaching of a third-grade mathematics lesson. She explains that she was showing a group of educators a segment from her lesson on odd and even numbers for her third-grade class. The video segment began with a student, Sean, proclaiming that he had been thinking about how the number six could be both odd and even because it was made of "three two's" and "two three's." Sean illustrated both scenarios on the board for his classmates and teacher to inspect. The other students challenged his conjecture of six being both an odd and even number and much talk was generated about it. In showing this video to educators, Ball hoped to generate a lively discussion on various ways this situation could have been handled, such as clarifying the definition of even and odd numbers or asking for other student's opinions. The educators watching the video immediately wanted to know if Ball used manipulatives or any concrete materials to clarify the meaning of odd and even numbers to Sean. When she explained that drawings and illustrations were used, the teachers became fiercely adamant that had Ball used physical counters, she could have more firmly guided her students toward the correct conclusion. Ball points out that, as a teacher, she does not want to prevent this sort of "discovery learning" that her students made in allowing them grapple with the ideas behind the math concepts of odd and even numbers. Ball also states that she is not convinced that allowing students to use manipulatives would automatically guide them to the correct mathematical conclusions. She claims that there is a common misconception among educators of a tremendous faith in the almost magical power of manipulatives to automatically convey the underlying mathematical knowledge with their mere presence. Ball asserts that it is the context with which the manipulatives are used that creates meaning, such as talk and interaction between teacher and students that evolves during the course of instruction. Ball claims that current education reform implies, in many ways, that manipulatives or physical materials are crucial in improving mathematics learning. This sentiment is reinforced in a number of ways such as the inclusion of manipulatives in mathematics curricula from school districts and publishers and the inclusion of "manipulative kits" to districts and schools that purchase their curriculum materials. The offering of in-service workshops and professional developments on manipulatives are also popular and sometimes required by school administration or districts. Ball asserts that there is not enough examination as to the validation of the appropriate role in helping students learn mathematics using manipulatives. Little discussion occurs as to possible uses of different types of concrete materials or possibly illustrations. It is assumed that students will "magically" learn the math concept and draw the correct conclusion that the teacher intended her students to derive from the activity. Ball claims that one of the reasons adults over emphasize the power of concrete representations to convey accurate mathematical skills is because adults are seeing concepts they already understand. Students who do not already possess this knowledge may not come to the same, correct conclusions about the underlying mathematical knowledge the manipulative is alleged to convey. Ball suggests that there is a need to examine the difficult problem of helping students make correct connections between the manipulative and the knowledge it is meant to convey. She discusses the need for teachers to develop rules for students as to how to operate with the manipulatives so that they are more likely to arrive at the correct mathematical conclusions. One such rule is when students are using base-ten blocks to subtract two-digit numbers with regrouping, they would have to trade in a ten bar for ten ones, in order to complete the correct regrouping procedure. A teacher could take that a step further and have the student relate this activity to the subtraction algorithm of regrouping and structure student talk and interaction that requires reflection, around the subtraction activity.

In Baroody's (1989) paper titled "Manipulatives Don't Come with Guarantees," the author contends that manipulatives are neither sufficient nor necessary for meaningful learning in mathematics. He acknowledges that they can be useful tools to students, however he discourages their "uncritical" use. Unless they are used thoughtfully there is no guarantee for meaningful learning; thoughtful use is essential in their effectiveness. Thoughtful use can be determined with questions such as: "Can the knowledge students gain from the use of this manipulative connect to their existing knowledge or be meaningful to them?" or, "Is the manipulative being used in a way that requires reflection or thought on the student's part?" Many times the answer to these questions is "no". In examining why manipulatives alone are not enough to guarantee meaningful learning, we need to discover what would make them enough. In other words, what do teachers need to do to make the manipulatives effective in conveying the underlying mathematical concept?

A 21-week qualitative pilot study conducted by Golafshani (2013) examined the practices of four 9th grade applied mathematics teachers concerning their beliefs about the use of manipulatives in teaching mathematics, its effects on learning and enabling and disabling factors. The teachers taught various topics to 9th grade students with diverse learning abilities. The teachers were given support, such as manipulatives, a math literacy tool kit, the opportunity for professional learning, training and dialogue, and resources to plan for five math lessons with manipulatives.

After each lesson, pre- and post-lesson discussions were provided. In the pre-lesson discussions, the teachers would describe the lessons, referencing the goals and manipulatives used. In the post-test discussion, the teacher would reflect on the lesson and discusses any modifications to the lesson that were needed to achieve greater student learning. A teacher questionnaire and an observation sheet were developed to collect data pre- and post-lesson data. Data were also collected through observations of the teacher's use, availability, and comfort level in utilizing manipulatives in the classroom. Teachers' beliefs about the use of manipulatives are important factors that could contribute to their effective use of manipulatives during instructional lessons.

When comparing pre- and post-test findings of teachers' views about teaching with manipulatives, teachers showed more interest in the use of manipulatives in the post-test. This could have been attributed to the fact that the teachers now had more confidence in their use of manipulatives during their instructional lessons. Teachers also showed strong agreement concerning student learning with manipulatives pre- and post-lessons. Some of the disabling factors teachers identified in the pre-test in the implementation of manipulatives were lack of confidence and lack of time to practice. These factors were not identified as disabling in the post-test, which was possibly due to the confidence they gained in the use of manipulatives during the pilot study. Factors identified as disabling in teaching with manipulatives during the post-test were lack of time to prepare and lack of knowledge of multiple uses of certain manipulatives. The identification of these disabling factors by teachers in the post-test might be due to the training which made them knowledgeable enough on the topic to realize the time it takes to prepare for a lesson with manipulatives, and that there may be other uses with the same manipulative. Teachers identified "difficulty with classroom management" both pre- and post-test, which could show a need for training or support in this area.

The diagram below shows that at the core of all disabling and enabling factors within the Golafshani study, existed the need for teacher training. The most significant relationship to teacher training is administrative support. Without administrative support, the training of teachers would which made them knowledgeable enough on the topic to realize the time it takes to prepare for a lesson with manipulatives, and that there may be other uses with the same manipulative. Teachers identified “difficulty with classroom management” both pre- and post-test, which could show a need for training or support in this area.

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The diagram below (See Figure 1) shows that at the core of all disabling and enabling factors within the Golafshani study, existed the need for teacher training. The most significant relationship to teacher training is administrative support. Without administrative support, the training of teachers would be hindered (Golafshani, 2013). Enabling factors were classroom, time and space management

Teachers' Beliefs and Teaching Mathematics with Manipulatives

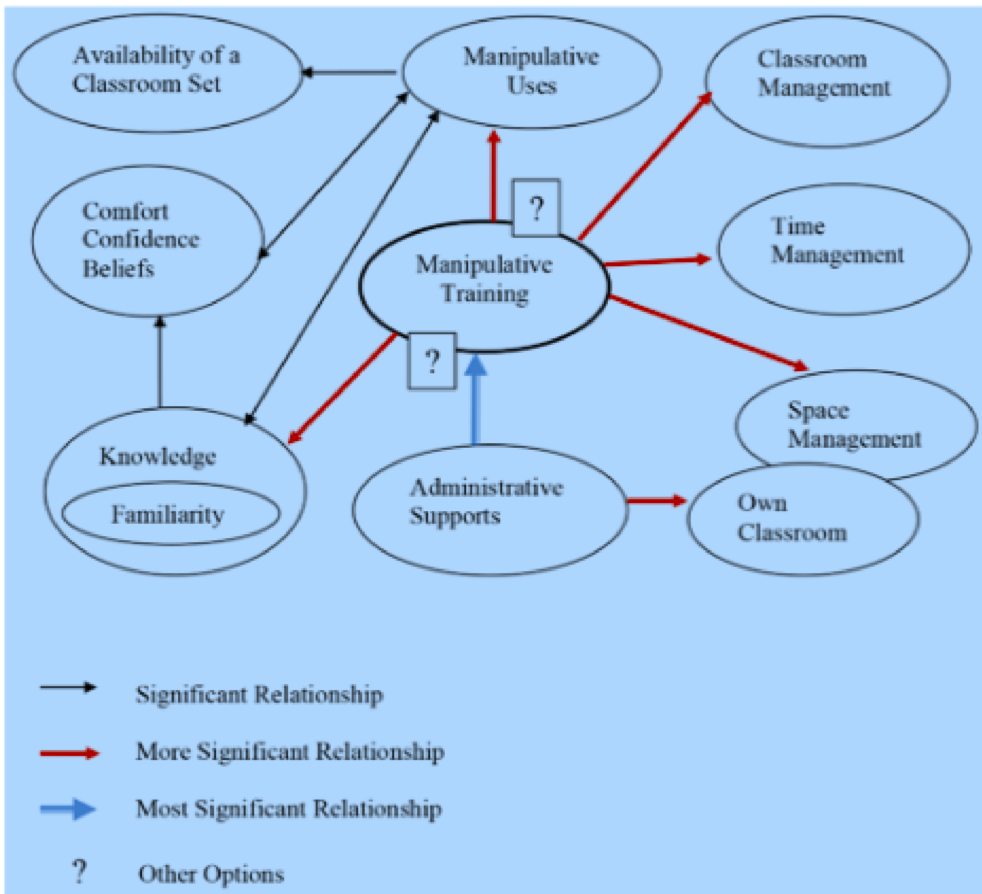


Figure 1: The Links Among the Disabling Factors, Enabling Factors and the Themes

factors teachers identified in both the pre-and post-test along with the availability of manipulatives, support from administration, and training on how and when to use manipulatives. Studies show that there is a strong association between teachers' knowledge and teachers' confidence, especially for those who are committed to constructivist teaching (Ross-Hogaaboam-Gray & Hannay, 1999), which proved to be true in this particular study. It can be concluded from this study that teachers' beliefs can be influenced by a number of variables including training and administrative support.

A year-long study conducted by Moyer and Jones (2004) looked at how teachers used manipulatives in their classroom and students' usage of manipulatives in relationship to their teacher's instruction. It specifically examined how students reacted to "free access," or choice of how and when to use manipulatives. It included 10 middle grades math teachers who had participated in a 2-week summer institute, examining various methods of representations for teaching mathematics with conceptual understanding. The teachers were grouped into control or autonomy oriented groups, according to their classroom management styles. The study was implemented in three phases: pre-assessment, phase one, and phase two. In the pre-assessment phase, teachers were interviewed to identify background information, uses of math manipulatives, and instructional practices. In phase one, the teachers used manipulatives for their mathematics instructional lessons. In phase two, the teachers provided students with "free access" to manipulatives, located in containers on students' desks.

There was a significant difference between control-oriented and autonomy-oriented teachers following their summer training institute. Control-oriented teachers initially exerted more control over their students in phase one and then less control over their students in phase two. In comparison, the autonomy-oriented teachers exerted less control over their students in phase one as well as less control over their students in phase two. When allowed, students were found to use manipulatives appropriately and selectively for mathematics tasks and as a way to self-review previously taught material during the "free access" phase.

Some teachers in this study used control strategies to undermine student choice and discourage students "free access" to manipulatives. This inhibits the alignment of student and teacher thinking. This may be more comfortable to teachers who are not confident in their teaching abilities or usage of manipulatives and who do not want students challenging their ways of problem solving. This limits students' thinking to the teacher's line of thinking and discourages students from challenging the teacher's methods.

Teachers' beliefs about manipulative usage and comfort level with them play an important role in student access and manipulative usage in instructional lessons. Teachers limited student access by displaying lists on containers, assigning manipulative monitors, and using them as a reward/punishment tool. A limitation of this study was the teachers in the study were selected from a group of teachers attending a math summer institute workshop. They were not selected from a general pool of math educators, so it could be concluded that these teachers were more interested in student learning than the average teacher.

Uribe-Flórez and Wilkins (2010) examined 503 in-service elementary teachers' background characteristics, beliefs about manipulatives, and the frequency with which they used manipulatives as part of their instruction. The teachers were part of a professional development experience in which they were asked to complete a survey related to their beliefs, attitudes, and instructional practices associated with mathematics teaching.

Results showed that teachers thought manipulative usage was more important at the lower elementary grade levels than the higher elementary grade levels. This demonstrates the belief of teachers in this study, which is that a grade 3-5 teacher tends not to believe in the importance of having children participate in hand-on activities, contrasted with a Kindergarten teacher's belief that it is important for children to participate in hands-on activities.

Using a one-way analysis of variance (ANOVA) to investigate possible differences in teacher's manipulative use by grade level, researchers uncovered a significant difference in teachers' use of manipulative use by grade-level groupings. Kindergarten teachers were found to use manipulatives most often. The next most frequent manipulative usage came from first and second grade teachers. Third through fifth grade teachers used manipulatives least often. The teachers' age and experience teaching were related to manipulative usage when they were considered alone, although after controlling for teacher grade and beliefs, they were no longer statistically significant predictors of manipulative usage. This manipulative usage showed that teachers' beliefs play an important role in manipulative usage and is consistent with previous findings by Gilbert and Bush (1988) showing less use of manipulatives by teachers in higher grades than lower grades levels, at the elementary level.

Another study by Moyer (2001) was conducted over the course of one academic year in 2001, which looked at how teachers use manipulatives to teach mathematics. The 10 teachers involved in this study were selected from a group of 18 middle grades teachers enrolled in a summer math institute, where they received instruction in the use of manipulatives. The researchers used interviews and observations over the year to discover how and why teachers used manipulatives the way they do. During the interview portion of the study, teachers conveyed different beliefs for using manipulatives in their classrooms such as: change of pace, a reward or privilege, a visual model for introducing concepts, providing reinforcement or enrichment, and "a way to make it more fun." Teachers seemed to distinguish between "real math" and "fun math." "Real math" referred to lessons where they taught rules, procedures, and algorithms to their students through textbooks and "fun math" was used when teachers described parts of their lesson where students were utilizing manipulatives.

Teachers seemed to be conveying that manipulatives were fun, but not necessary for teaching and learning mathematics. They seemed to distinguish manipulative use from their "regular mathematics teaching." During classroom observations one teacher was observed telling a student, who had requested to use manipulatives to solve a problem, to solve it first mathematically, without the use of manipulatives. Numerous other comments were made by teachers indicating their dissatisfaction with the use of math manipulatives. Looking at the amount of time spent with manipulatives during daily math lessons that were observed provided a range from no use of manipulatives to 31 minutes of use. In the 40 lessons observed, students used manipulatives 7.38 minutes for every 57.5 minutes of math class time. Math manipulative usage accounted for approximately 13% of the math time.

Themes about Manipulative Usage

The overwhelmingly common theme in the research on Teacher's Usage of Math Manipulatives is the impact of teacher's beliefs on their teaching practices. This is the deciding factor in many instructional decisions made by teachers on a daily basis. Teachers' beliefs were evident when they showed a more positive attitude toward using manipulatives in their instructional lessons after training. Teacher training and support tends to foster a more positive attitude toward the use of manipulatives. This is an encouraging sign because it shows that teachers' beliefs are not so rigid and can be influenced.

Teachers who believe manipulatives are just used for change of pace, reward or privilege, or fun are not going to genuinely incorporate manipulatives and the concepts they were meant to convey into their instructional lessons. They are also sending a message to their students that manipulatives are similar to toys and are just meant for fun. The entire mathematical concept that the manipulative was meant to convey would be lost on these students.

Teachers' beliefs were evident when they restricted students from "free access" to the manipulatives by way of displaying lists on containers, assigning manipulative monitors, and using them as a reward/punishment tools. Teachers may feel threatened by this new learning environment. They would no longer be the “all-knowledgeable” person that students look to for the correct answer. Students may even discover new ways of solving math problems that challenge the teacher’s way of thinking. Teachers may not be comfortable with this new role and type of flexible thinking.

According Uribe-Flórez and Wilkins (2010), understanding the relationship between teachers’ beliefs about mathematics and teaching practices has been the focus of many studies. A study by Wilkins (2008) of 481 elementary teachers found that teachers’ beliefs were the most significant forecaster of teaching practices among other factors considered, such as content knowledge and attitudes. Research that distinctly looks at teachers’ beliefs in conjunction with manipulative use has proved to be inconclusive according to Moyer (2001), and others.

Many of these studies charge that teachers’ beliefs are paramount to the effective use of manipulatives in the classroom, and that administrative support, trainings, and other factors can influence their beliefs. Even though these factors are imperative to effective manipulative use by teachers, I believe there is another, even more powerful factor that influences teachers’ effective use of manipulatives in instructional lessons. That factor is cognitive guidance.

Cognitive guidance occurs when teachers elicit and guide students’ mathematical thinking to help them make connections to existing knowledge, in order to encourage deep conceptual understanding. This is similar to instructional guidance, which is one of the most commonly examined factors in educational research. However, instructional guidance pertaining specifically to manipulatives has limited available research.

The effectiveness of instructional guidance is contradictory (Kuhn, 2007). However, a recent meta-analysis by Alfieri et al. (2011) found that unassisted discovery does not benefit learners, whereas high guidance and elicited explanations do. Marley and Carbonneau (2014) assert that rather than determining if instruction with manipulatives is more effective than conventional instruction, more effort should be made to examine the value added by various instructional factors that may accompany instruction with manipulatives.

All of this aligns with Ball (1992), who stated that there is not enough examination as to the validation of the appropriate role in helping students learn mathematics using manipulatives. It would likely be more helpful to teachers if more professional development opportunities were made available that specifically focused on teachers learning to help students make the important connections between the mathematics manipulatives and the underlying mathematics concepts they are investigating and how they can be used within instruction. Marilyn Burns (n.d.) has been an advocate for math manipulatives now for over 30 years. This lead to her company *Math Solutions* which provides resources and training on using math manipulatives to educators around the world. Her role today in the math manipulative movement has been far reaching and she provides videos and demos of the most common manipulatives used today through demonstration of using the manipulative to teach most math concepts teachers need to cover in today's math classrooms.

Common Mathematics Manipulatives and Their Uses

It is important that today's math teachers use math manipulatives to make math concepts concrete rather than abstract. Teachers can obtain commercial-made manipulatives, make their own, or help the students make their own. Examples of manipulatives are paper money, buttons, blocks, Cuisenaire rods, tangrams, geoboards, pattern blocks, algebra tiles, and base-ten blocks. The use of manipulatives (See Figure 2 and Table 1 for examples) provides teachers with a great potential to use their creativity to do further work on the math concepts instead of merely relying on worksheets. Consequently, students learn math in an enjoyable way, making connections between the concrete and the abstract. Piaget and Montessori philosophies are still alive and well received in today's math classroom. The CRA (Concrete-Representational-Abstract) Model for teaching mathematics is the main approach for teaching most math concepts for K-8 learners. When teaching mathematics, teachers always need to start with concrete manipulative materials to first teach for understanding, then transfer to representational models like pictures or diagrams, leading and bridging learning to the abstract level of understanding of symbols and operation signs so that students eventually do not need the manipulatives to do the mathematics.

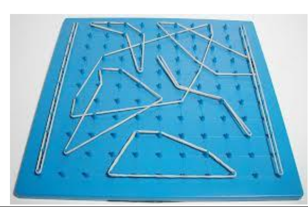
Figure 2: Common Math Manipulatives in US Schools

To understand the concept of money, teachers can have students "buy" items tagged for sale in the classroom. Students are given an opportunity to describe purchases they or an adult have made. Students select the proper combinations of coins to purchase the item. As each student participates, the class helps by showing the coins on the overhead. By handling the coins, students can correct mistakes and verify counting amounts of money.

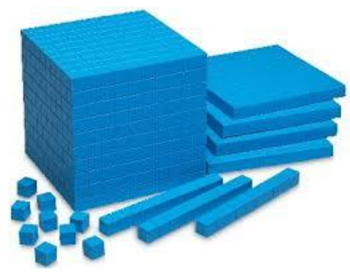
Many studies over the years have demonstrated the benefits of using multiple modalities. English Language Learners (ELL) students, however, are disadvantaged in the one modality teachers seem to use the most: auditory. Claire and Haynes (1994) stated,

> Of the three learning modes—auditory, visual, and kinesthetic—ESOL students will be weakest in auditory learning. It is unrealistic to expect them to listen to incomprehensible language for more than a few minutes before tuning out. But if you provide illustrations, dramatic gestures, actions, emotions, voice variety, blackboard sketches, photos, demonstrations, or hands-on materials, that same newcomer can direct his or her attention continuously. (p. 22)

Manipulatives are powerful tools and can be used to teach many of the new *Common Core State Standards (CCSS) in Mathematics* (National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO), 2010), as the following chart shows:

Manipulative	**Common Core Math Standard Covered**	**Image of Manipulative**
Geoboards	***CCSS.Math.Content.3.MD.C.5*** Recognize area as an attribute of plane figures and understand concepts of area measurement.	

Pattern Blocks	***CCSS.Math.Content.K.G.A.3*** Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).	
Tangrams	***CCSS.Math.Content.1.G.A.1*** Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	
Color Tiles	***CCSS.Math.Content.2.G.A.2*** Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	
Unifix/Snap Cubes	***CCSS.Math.Content.5.MD.C.3*** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. ***CCSS.Math.Content.5.MD.C.3a*** A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. ***CCSS.Math.Content.5.MD.C.3b*** A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.	
Triman Compass	***CCSS.Math.Content.4.G.A.1*** Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	

		
Cuisenaire Rods	***CCSS.Math.Content.7.RP.A.1*** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½/1/4 miles per hour, equivalently 2 miles per hour.	
Base-10 Blocks	***CCSS.Math.Content.1.NBT.B.2*** Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: ***CCSS.Math.Content.1.NBT.B.2a*** 10 can be thought of as a bundle of ten ones — called a "ten." ***CCSS.Math.Content.1.NBT.B.2b*** The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. ***CCSS.Math.Content.1.NBT.B.2c*** The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	
Number Tiles	***CCSS.Math.Content.K.CC.A.3*** Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).	

TI Explorer Plus Calc.	***CCSS.Math.Content.8.EE.A.4*** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	
Two-sided Counters	***CCSS.Math.Content.6.NS.C.5*** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	
Judy Clock	***CCSS.Math.Content.1.MD.B.3*** Tell and write time in hours and half-hours using analog and digital clocks.	
Abacus	***CCSS.Math.Content.1.NBT.C.4*** Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	
	CCSS.Math.Content.6.EE.A.4	

Scale/ Balance	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.	

Table 1: Chart of Math Manipulatives with CCSS in Math and Image

Summary

Teachers need to learn how to encourage student exploration, related discussion, and reflection about the prospective math concept they teach. They need to be comfortable with students' exploration of the math concepts and possibly wandering off the "correct" track or even challenging the teachers' own mathematical viewpoint. Teachers cannot assume that when students use manipulatives they will automatically draw the correct conclusions from them. Adults may overestimate the power of manipulatives because they already understand the underlying math concepts that are being conveyed by the math manipulatives. Teachers need to keep in mind that the student does not already possess this knowledge and still needs to make the correct connections between the manipulative and the underlying math concept. While math manipulatives are a valuable tool in the instruction of mathematics, teachers need to bridge the manipulatives to the representational and then abstract understanding in mathematics so that students internalize their understanding. Just using manipulatives by themselves without this may not have great value. Today, in an age of technology and high-stakes testing, teachers need to use and bridge the gap for students in using math manipulatives. This then can be connected to representational and abstract ideas in mathematics to help students deeply understand the math they are learning and needing to apply to our everyday life.

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