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Experiment 2.04: Kirchhoff's Laws

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I. EXPERIMENT 2.04: KIRCHHOFF'S LAWS

A. Abstract

The equivalent resistance of a resistor arrangement that is not reducible by series and parallel analysis is determined experimentally. Some of the measurements are compared to results based on Kirchhoff's method.

B. Formulas

$$\text{KVL: } \sum_{\text{loop}} \Delta V = 0 \tag{1}$$

$$\text{KCL: } \sum I_{\text{in}} = \sum I_{\text{out}} \text{ at junctions} \tag{2}$$

Resistor color codes:

color	no.
black	0
brown	1
red	2
orange	3
yellow	4
green	5
blue	6
violet	7
gray	8
white	9

$$R = (10c_1 + c_2) \times 10^{c_3}$$

where $(c_1\ c_2\ c_3)$ represents the resistor's 3-color bands and R is the value of the resistance in Ohms.

C. Description and Background

While Ohm's Law can be used to derive straightforward equations to calculate the resistance, potential, and current of simple series and parallel arrangements, what happens when a resistor is both in series and parallel at the same time?

This experiment investigates a complex arrangement of resistors using two laws first studied by Gustav Kirchhoff in 1845:

The Loop Rule: The algebraic sum of changes in potential around any closed loop in a circuit must be zero. In other words, if you start at some potential, then walk around a closed loop, you must be back where you started at the end of the loop (the change will be zero)

$$\sum_{\text{loop}} \Delta V = 0$$

The Junction Rule: The sum of all currents entering a junction must equal the sum of all currents leaving the junction. What goes in must come out

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

Combined, Kirchhoff's Laws can be used to write a system of equations that will allow you to solve for each of the unknown quantities in the circuit.

In the schematic shown in Figure 1, five resistors are arranged in an intertwined configuration. Different currents flow in each branch of the circuit and are labeled as such. Including the current, I, which flows into and out of the arrangement, this results in six different currents that must be determined.

D. Procedure

1. Measure and record the resistances of the five resistors used using the ohmmeter. See Appendix I at the end of this document for important instructions on the use of multimeters.

2. Measure the resistance (R_{Ohmmeter}) of the resistor arrangement using the ohmmeter.

3. Set up the circuit on the circuit board according to the diagrams that follow on the next page.

4. Measure the *terminal voltage of the battery* when connected to the resistor arrangement, and record this as the emf.

5. Measure the currents, I_1, I_2, I_3, I_4, I_5, identified in Fig. 1.

1. *Figures*

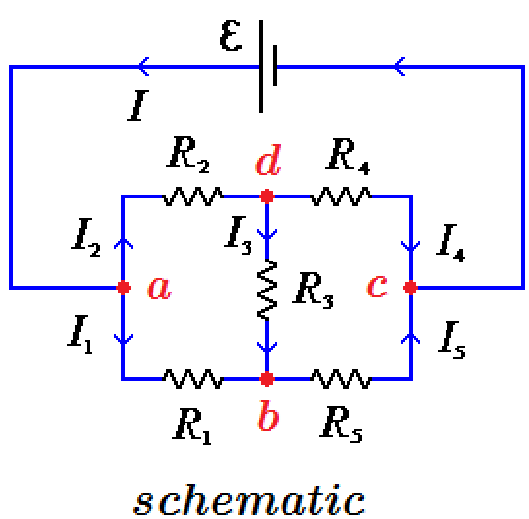

FIG. 1. Circuit used in the experiment.

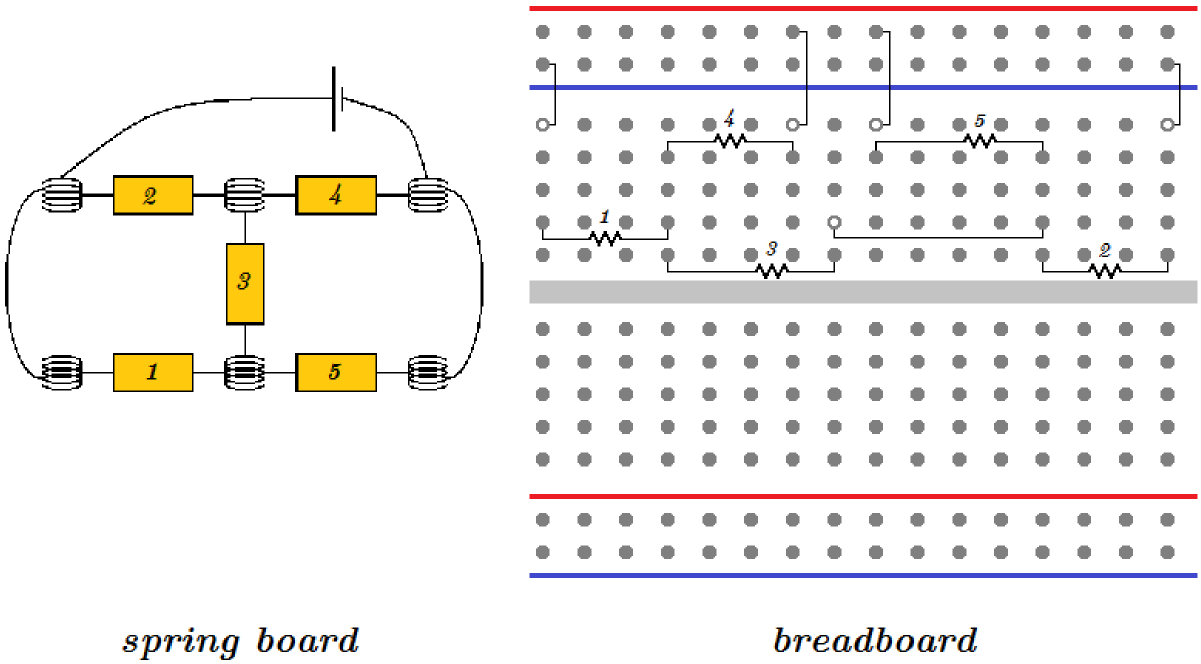

FIG. 2. Kirchhoff's Laws Arrangement

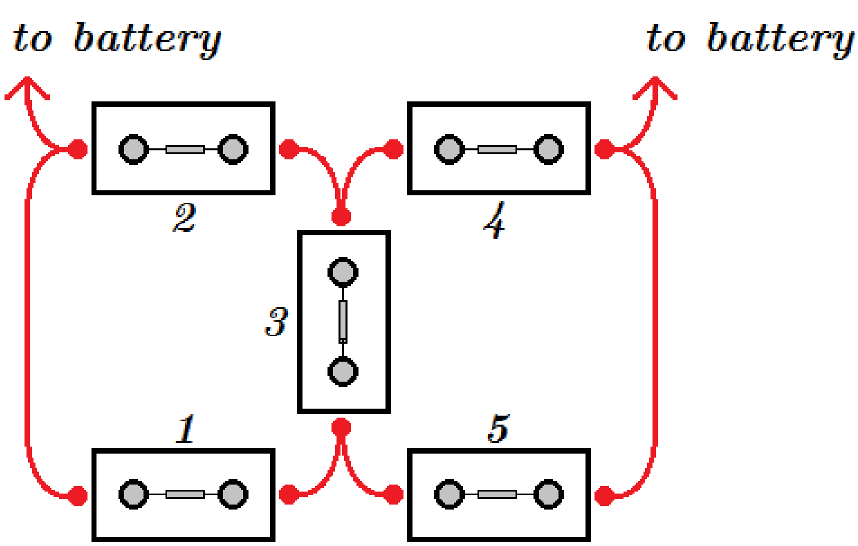

FIG. 3. Kirchhoff's Laws: Di-Binding-Post Blocks

E. Measurements

$\mathcal{E}$ [$Volt$]	

segment	R [Ω]	I [mA]
1		
2		
3		
4		
5		

measured resistance of the arrangement, R_{Ohmmeter} [Ω]	

F. Instructions

1. Calculate the current drawn from the emf using the measured currents

$$I = \frac{1}{2}\left(I_1 + I_2 + I_4 + I_5\right)$$

2. Calculate the resistance of the arrangement using

$$R_{\text{calculated}} = \frac{\mathcal{E}}{I} \tag{3}$$

3. Calculate the percent difference between the calculated value above and the measured value, R_{Ohmmeter}.

4. Using Kirchhoff's laws, the circuit used in this experiment can be analyzed and the following formulas derived for the currents I_2 and I_4

$$I_2 = \left[R_1R_3 + R_1R_4 + R_1R_5 + R_3R_5\right]\ J \tag{4}$$

$$I_4 = \left[R_1R_3 + R_1R_5 + R_2R_5 + R_3R_5\right]\ J \tag{5}$$

where

$$J = \frac{\mathcal{E}}{R_1R_2R_3 + R_1R_2R_4 + R_1R_2R_5 + R_1R_3R_4 + R_1R_4R_5 + R_2R_3R_5 + R_2R_4R_5 + R_3R_4R_5} \tag{6}$$

5. Use the junction rule at junction d to determine I_3.

6. Use the loop rule on the loop abd to determine I_1.

7. Use the junction rule at junction b to determine I_5.

8. Using Kirchhoff's laws, the circuit used in this experiment can be analyzed and the following formula derived for the equivalent resistance of this arrangement

$$R_{\text{Kirchhoff}} = \frac{R_1R_2R_3 + R_1R_2R_4 + R_1R_2R_5 + R_1R_3R_4 + R_1R_4R_5 + R_2R_3R_5 + R_2R_4R_5 + R_3R_4R_5}{R_1R_3 + R_1R_4 + R_1R_5 + R_2R_3 + R_2R_4 + R_2R_5 + R_3R_4 + R_3R_5}$$

Use this equation to calculate the equivalent resistance theoretically.

G. Calculations

$I = (I_1 + I_2 + I_4 + I_5)/2$ [mA]	
$R_{\text{calculated}}$ [Ω]	
%-Diff $(R_{\text{Ohmmeter}}, R_{\text{calculated}})$	
I_2 [mA]	
I_4 [mA]	
I_3 [mA]	
I_1 [mA]	
I_5 [mA]	
R_{Kirchoff} [Ω]	
%-Diff $(R_{\text{Ohmmeter}}, R_{\text{Kirchhoff}})$	

II. APPENDIX I

A. Multimeters

Multimeters are versatile electronic devices that can be used to measure properties such as voltage, current, and resistance. Multimeters typically employ two leads, one of which is always plugged into a socket labelled COM for common. The second lead is plugged into the socket appropriate to the function desired and usually labelled V (Volts) for voltage, mA, μA, or 10A (Amps) for current (depending on the expected range), and Ω (Ohms) for resistance. There is also a dial that turns the multimeter on and must be turned to the appropriate setting. There is a toggle key that will switch from DC to AC mode and a toggle key that can switch the numerical range of measurements.

B. Voltmeters

Voltmeters are nonintrusive to a circuit. The leads are used to probe the voltage between two points in the circuit. When using a voltmeter to measure the voltage across a circuit element, attach the leads (alligator clips) to both ends of the element under consideration.

C. Ammeters

Ammeters, unlike voltmeters, are intrusive to a circuit in that they must be inserted into the circuit segment whose current is to be measured. The circuit segment must therefore be broken and the ammeter placed in series with the other segment elements.

When using the ammeter with the circuit spring board, lift one end of the resistor from the spring it is attached to and bridge the gap with the ammeter leads, *i.e.*, attach one lead to the spring and the other lead to the free end of the resistor.

When using the ammeter with the circuit breadboard, lift the end of the jumper wire depicted in the figures (*e.g.,* those in Experiments 2.02-2.04) by an open circle. Attach one lead to the free end of the jumper wire and the other lead to the exposed wire of the resistor on the same terminal strip (column in the figures).

The circuit should remain open until the ammeter is inserted. Once properly inserted, the ammeter is turned on. The circuit is then closed, and the current measurement taken.

Removal of the ammeter should proceed in reverse: The circuit is opened, then the ammeter is turned off and removed. *Failure to follow this procedure may result in damage to the ammeter.*

D. Ohmmeters

An ohmmeter measures the resistance of whatever is connected between its two leads. Consequently, it is important that only those elements (or arrangement), whose resistance is sought, be positioned between the leads. This may require separating the arrangement of elements from the rest of the circuit to make the proper ohmmeter measurement. Case (a) in the figure below depicts an example of a faulty ohmmeter measurement. The goal is to measure R_1, but this resistor is not the only element between the ohmmeter leads in case (a). The reading in case (a) would actually be $R_1\,(R_2 + R_3)\,/\,(R_1 + R_2 + R_3)$. In case (b) the resistor of interest is separated out and is the only element between the leads.

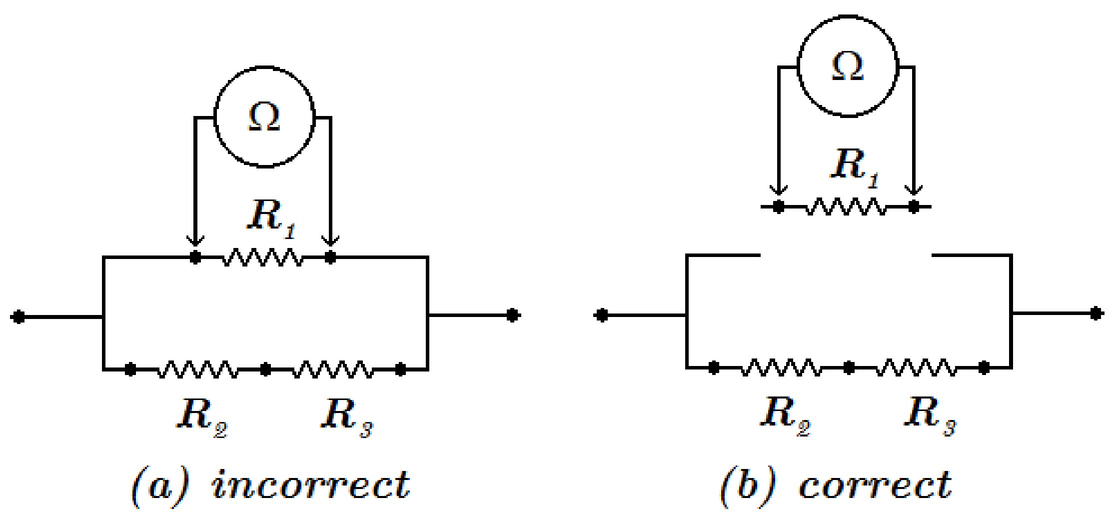

FIG. 4. Example of improper and proper use of ohmmeter.