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Number Pattern

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Cover Page Footnote
This article is the result of the MAT students' collaborative research work in the Pre-Algebra course. The research was under the direction of their professor, Dr. Hui Fang Su. The paper was organized by Team Leader Denise Gates.

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Number Patterns

Abstract

In this manuscript, we study the purpose of number patterns, a brief history of number patterns, and classroom uses for number patterns and magic squares. We investigate and summarize number patterns and magic squares in various charts: 6 x 6, 7 x 7, 13 x 13, 21 x 21, and 37 x 37. The results are established by each number pattern along with narrative conjectures about primes and multiples of six from each pattern. Numerical charting examples are provided as an illustration of the theoretical results.

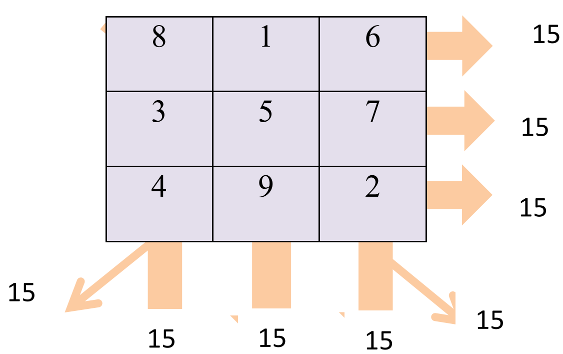
A Brief History of Number Patterns

Recognizing number patterns is a vital problem-solving skill. As noted by the Annenburg Foundation, “If you see a pattern when you look systematically at specific examples, you can use that pattern to generalize what you see into a broader solution to a problem” (Annenburg Foundation, 2016). Understanding number patterns are necessary so that students of all ages can appropriately identify and understand various types of patterns and functional relationships. Furthermore, number pattern awareness allows one to use patterns and models to analyze the change in both real and abstract contexts. The Common Core State Standards state that “mathematically proficient students look closely and discern a pattern or structure” (CCSS, 2015). In addition to the Common Core State Standards, the National Council of Teachers of Mathematics states that “In prekindergarten through grade 2 all students should use multiple models to develop initial understandings of place value and the base-ten number system” (NCTM, 2015).

How can number sense and number pattern awareness be developed and or enhanced upon? A hundreds chart can be used to provide students with a framework for thinking about the base ten number system; it allows students to develop a mental model of the number system. Utilizing a hundreds chart will provide students with the opportunity to look for and make sense of number patterns and structure within the base ten number system. Furthermore, the familiarity with a hundreds chart will also build upon a student's sense of number patterns and awareness, and, therefore, lead to computational flexibility and fluency. A hundreds chart can be used for a variety of activities. Using a hundreds chart, students can look for number patterns, they can brush up on their addition and subtraction facts, they can build their multiplication sense, they can broaden their knowledge of fractions and decimals and enhance their logical and strategic thinking skills (Gaskins, 2008).

A Brief History of Magic Squares

One of the oldest and most revered puzzles of all time is the magic square puzzle. A magic square is formed by arranging consecutive numbers in a square so that the rows, columns, and diagonals add equally. For example, a 3x3 magic square could arrange with the numbers 1 through 9 so each row, column, and diagonal adds to 15:

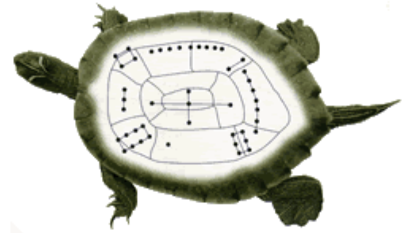


There are several other ways to array the numbers in each of the nine cells to create a 3x3 magic square. Given the task of arranging consecutive numbers in a pattern so that the rows and columns form an equal sum, one can add the consecutive numbers and divide by the number of columns or rows. In the example above, 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45 / 3 = 15 are the sum of the rows, columns, and diagonals in the magic square. It can be an exercise to the student – if the student can determine the sum established on the number of rows and columns in the square, he or she can arrange consecutive numbers in the square appropriately. The larger the square, the more cumbersome this methodology becomes, which is why mathematicians have created the following formula:

$$M = \frac{n(n^2+1)}{2}.$$

In the above formula, n symbolizes the number of rows and columns. If we continue to use the example of a 3 x 3 square, the formula would simplify to $(3(3^2 + 1) / 2) = 15$. Understandably, the larger n becomes larger than the sum, thus creating possible arrangements throughout the square (Magic Squares - What are they).

The origin of the magic square can be traced back to 2800 B.C. Chinese Literature and the legend of "Lo Shu" which can be translated to the scroll of the River Lo. The legend tells the story of a flood that destroyed the land. The people of the city believed that offering a sacrifice to the river god would calm his anger and keep him from causing further destruction; however the river god continued to flood the land. The legend states that a turtle would emerge from the river after every flood and walk around the sacrifice. After this had happened several times, a child noticed a unique pattern on the turtle's shell. This pattern, which can be seen in the figure to the right, told the people how many sacrifices to make for the river god to calm the waters – fifteen. Other accounts of this tale state that it was the great Emperor Yu



Fiq 2. Lo Shu, the oldest known magic square

who was the one to notice this pattern –whoever it was is unclear, but each account does state that the fifteen sacrifices pleased the river god, and the flooding was ceased (Magic Squares - What are they).

Early Chinese culture is rich with the usage of magic squares, including astrology, divination, philosophy, natural phenomena, and human behavior. Magic squares and their uses in various areas of study traveled from China to other parts of Asia and Europe throughout ancient civilization. Its history is rich, and the square has journeyed a long period. Some of its greatest achievements in India's history include Varahamihira, who used a fourth-order magic square to specify recipes for making perfumes that allowed him to see into the future; and the doctor Vrnda, who claimed the magic square helped him develop a means to ease childbirth (Anderson).

The ancient Arabic description for magic squares, wafq ala'dad, means "harmonious disposition of the numbers." The idea is exemplified by Camman, who speaks of the spiritual importance of these magical puzzles:

> "If magic squares were, in general, small models of the Universe, now they could be viewed as symbolic representations of Life in a process of constant flux, constantly being renewed through contact with a divine source at the center of the cosmos." (Prussin 1986, p. 75)

Much of ancient history reveals continued to awe and reverence for the magic square – Ancient artifacts from Africa to Asia show that the magic square became interwoven into cultural artifacts, appearing on antique porcelains and sculptures, even in the design and building of homes (Anderson).

It continued throughout history until the seventeenth century, when French aristocrat Antoine de la Loubere began to study the mathematical theory behind the construction of magic squares. In 1686, Adamas Kochansky created a three-dimensional magic square. Today, magic squares are examined about factor analysis, combinational mathematics, matrices, modular arithmetic, and geometry (Anderson).

6 x 6 Chart – Number Pattern

During number pattern observation, it is highlighted that number patterns are evident. These are some indication:

Mult of 6

1	2	3	4	5	**6**
7	8	9	10	11	**12**
13	14	15	16	17	**18**
19	20	21	22	23	**24**
25	26	27	28	29	**30**
31	32	33	34	35	**36**

Mult of 4

1	**2**	3	**4**	5	**6**
7	**8**	9	**10**	11	**12**
13	**14**	15	**16**	17	**18**
19	**20**	21	**22**	23	**24**
25	**26**	27	**28**	29	**30**
31	**32**	33	**34**	35	**36**

Mult of 3

1	2	**3**	4	5	**6**
7	8	**9**	10	11	**12**
13	14	**15**	16	17	**18**
19	20	**21**	22	23	**24**
25	26	**27**	28	29	**30**
31	32	**33**	34	35	**36**

Primes

1	**2**	**3**	4	**5**	6
7	8	9	10	**11**	12
13	14	15	16	**17**	18
19	20	21	22	**23**	24
25	26	27	28	**29**	30
31	32	33	34	35	36

i. 2nd, 4th, and 6th columns are all divisible rule of 2 and multiples of 4, which indicates the multiple of 4 table.

ii. The table shows multiple of 6, which displays the last column numbers that are divisible by 6 and multiples of 6.

iii. The 3rd column numbers are divisible by 3, which is indicated in black.

iv. 1st, 3rd, and 5th columns are odd numbers, also the 2nd, 4th, and 6th columns are even numbers.

v. Each number arrangement shows the columns are increasing by 6.

vi. The table shows numbers are prime numbers on the 6 x 6 chart.

6 x 6 Chart – Number Pattern

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

During number pattern diagonal observation, looking at highlighted ray lines we can see patterns arrays:

vii. Looking at the above chart, the corners of the number arrangements on opposite ends, we can see a pattern of the sum of 37. For example, 6 + 31 = 37 and 1 + 36 = 37.

viii. Using orange ray lines in Figure 2, the diagonal will display a formula of n + 5. This pattern increased by five from top right to bottom left. For example, if we use n = 5, then the formula n + 5 is 5 + 5 = 10, which makes the next diagonal number arrangement 10. The pattern is the same for all diagonal numbers from top right to bottom left. However, if we reverse the indicated orange ray line, then we can see different formula n – 5 using the integer rule. The diagonal going upward from bottom left to right shows another pattern arrangement. If n = 13, then 13 – 5 = 7, which is the next diagonal number arrangement. The orange arrow indicates the pattern formulas n + 5 and n – 5; in fact, we can use it to demonstrate integer rules in a pattern observation.

ix. Looking at the above chart, the indicated blue arrow going diagonally will display a formula of n + 7 and n – 7, which demonstrate integer rules patterns observation. For example, if we use n = 36, then the formula 36 – 7 decrease by seven from going upward from bottom right to top left. The sum of the next diagonal number from top left to bottom right is 29. Also, if n = 13, then 13 + 7 = 20, which is the next diagonal number arrangement increased by 7. The pattern is the same for all diagonal numbers from top left to bottom right.

6 x 6 Chart – Magic Square

6x6 Magic Square

8	1	6	26	19	24
3	5	7	21	23	25
4	9	2	22	27	20
35	28	33	17	10	15
30	32	34	12	14	16
31	36	29	12	18	11

During number pattern observation, looking at the magic square we can see patterns arrays:

x. All rows, columns, and diagonals must add up to the magic square constant of 111 for a 6x6 board. The formula is $[n(n^2+1)]/2$.
The solution is $\frac{[6(6^2+1)]}{2}, \frac{6(37)}{2}, \frac{222}{2}, 111.$

7 x 7 Number Patterns

Mult of 6

1	2	3	4	5	**6**	7
8	9	10	11	**12**	13	14
15	16	17	**18**	19	20	21
22	23	**24**	25	26	27	28
29	**30**	31	32	33	34	35
36	37	38	39	40	41	**42**
43	44	45	46	47	**48**	49

Mult of 4

1	2	3	**4**	5	6	7
8	9	10	11	**12**	13	14
15	**16**	17	18	19	**20**	21
22	23	**24**	25	26	27	**28**
29	30	31	**32**	33	34	35
36	37	38	39	**40**	41	42
43	**44**	45	46	47	**48**	49

Mult of 3

1	2	**3**	4	5	**6**	7
8	**9**	10	11	**12**	13	14
15	16	17	**18**	19	20	**21**
22	23	**24**	25	26	**27**	28
29	**30**	31	32	**33**	34	35
36	37	38	**39**	40	41	**42**
43	44	**45**	46	47	**48**	49

Primes

1	**2**	**3**	4	**5**	6	**7**
8	9	10	**11**	12	**13**	14
15	16	**17**	18	**19**	20	21
22	**23**	24	25	26	27	28
29	30	**31**	32	33	34	35
36	**37**	38	39	40	**41**	42
43	44	45	46	**47**	48	49

7 x 7 Chart - Observations of the Number Patterns:

i. The values in the 7th (and last) column are all multiples of 7 and divisible by 7.

ii. The columns headed by an odd number alternate odd and even numbers until it reaches the bottom; the columns headed by an even number are exactly the opposite, alternating even and odd numbers until the bottom listed number.

iii. In each column, the numbers are increasing by seven from top to bottom.

iv. The table shows prime numbers in the 7x7.

v. The diagonal going from bottom left to the upper right is n-6, upper right to lower left n+6, The diagonal going from lower right to upper left is n-8, upper left to lower right is n+8.

vi. There is a congruency in the sum of the numbers at opposite ends of the grid:1+49=50 and 7+43=50.

7x7 Magic Square

30	39	48	1	10	19	28
38	47	7	9	18	27	29
46	6	8	17	26	35	37
5	14	16	25	34	36	45
13	15	24	33	42	44	4
21	23	32	41	43	3	12
22	31	40	49	2	11	20

7 x 7 Chart – Observation of the Magic Square:

vii. For a 7x7 grid, all rows, diagonals, and columns must add up to the magic number of 175; formula $[n(n^2+1)]/2$. The solution is $\frac{[7(7^2+1)]}{2}, \frac{7(50)}{2}, \frac{350}{2}, 175.$

viii. As noted with highlighting, every 7 numbers follows a diagonal pattern.

13 x 13 Chart - Number Patterns

The number 13 is unique in many ways, aside from being a very significant number and the seventh odd number. The number 13 it is also a part of one of the Pythagorean triples (13, 84, 85) (2010). Below is a 13 x 13 chart I constructed, starting with number 1. Given this information of the quantity and structure (using numbers 1 through 169, since 13x13=169), the conjecture would be that there is always, at least, one prime per row, and there is always, at least, two multiples of six per column.

Key:

Prime #s
Multiple of 6

1	2	3	4	5	6	7	8	9	10	11	12	13
14	15	16	17	18	19	20	21	22	23	24	25	26
27	28	29	30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49	50	51	52
53	54	55	56	57	58	59	60	61	62	63	64	65
66	67	68	69	70	71	72	73	74	75	76	77	78
79	80	81	82	83	84	85	86	87	88	89	90	91
92	93	94	95	96	97	98	99	100	101	102	103	104
105	106	107	108	109	110	111	112	113	114	115	116	117
118	119	120	121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140	141	142	143
144	145	146	147	148	149	150	151	152	153	154	155	156
157	158	159	160	161	162	163	164	165	166	167	168	169

Moreover, I have identified the prime numbers (pink) and multiples of six (blue). From the 13x13 chart, one can recognize that there are a few patterns throughout. For instance, there is a diagonal pattern with the numbers 66, 54, 42, 30, 18, and 6 since they are divisible by six and increase in odd increments when divided: 11, 9, 7, 5, 3, and 1. A similar pattern takes place in the first column row 12; beginning with 144, then 132 all the way to 24 and 12. This diagonal pattern also has multiples of six, but they are in increments of 12 (144/6=24, 132/6=22,…

24/6=4, and 12/6=2) and decreases by two every time. There is an additional correlation with the numbers 1, 13, and 169 in three corners of the chart as skew-related cells. Furthermore, two Pythagorean triangles were identified in the first row, using numbers: 3, 4, 5 and 5, 12, 13.

In 1963, Simon de la Loub’ere (1642-1729), a mathematician, recognized an algorithm to construct an odd order square. The pattern began with 1 in the central lower cell, and then continues diagonally upward to the right in the next column. The next digit, 3, is placed diagonally downward to the right of 2, and this continues for 3, 4, 5, 6, and 7. The remarkable chart below for the 13x13 pattern follows this format and can lucidly see the patterns of sequence organized in an arrangement of colors (Danesi).

93	108	123	138	153	168	1	16	31	46	61	76	91
107	122	137	152	167	13	15	30	45	60	75	90	92
121	136	151	166	12	14	29	44	59	74	89	104	106
135	150	165	11	26	28	43	58	73	88	103	105	120
149	164	10	25	27	42	57	72	87	102	117	119	134
163	9	24	39	41	56	71	86	101	116	118	133	148
8	23	38	40	55	70	85	100	115	130	132	147	162
22	37	52	54	69	84	99	114	129	131	146	161	7
36	51	53	68	83	98	113	128	143	145	160	6	21
50	65	67	82	97	112	127	142	144	159	5	20	35
64	66	81	96	111	126	141	156	158	4	19	34	49
78	80	95	110	125	140	155	157	3	18	33	48	63
79	94	109	124	139	154	169	2	17	32	47	62	77

Another mastermind puzzlist, who was a prison inmate at the time, created a 13x13 magic square with 11x11 and 3x3 nested inside. The Journal of Recreational Mathematics published this piece noting that each square is exactly 10,874 smaller than the last, and every cell is prime (Journal, 2010).

1153	8923	1093	9127	1327	9277	1063	9133	9661	1693	991	8887	8353
9967	8161	3253	2857	6823	2143	4447	8821	8713	8317	3001	3271	907
1831	8167	4093	7561	3631	3457	7573	3907	7411	3967	7333	2707	9043
9907	7687	7237	6367	4597	4723	6577	4513	4831	6451	3637	3187	967
1723	7753	2347	4603	5527	4993	5641	6073	4951	6271	8527	3121	9151
9421	2293	6763	4663	4657	9007	1861	5443	6217	6211	4111	8581	1453
2011	2683	6871	6547	5227	1873	5437	9001	5647	4327	4003	8191	8863
9403	8761	3877	4783	5851	5431	9013	1867	5023	6091	6997	2113	1471
1531	2137	7177	6673	5923	5881	5233	4801	5347	4201	3697	8737	9343
9643	2251	7027	4423	6277	6151	4297	6361	6043	4507	3847	8623	1231
1783	2311	3541	3313	7243	7417	3301	6967	3463	6907	6781	8563	9091
9787	7603	7621	8017	4051	8731	6427	2053	2161	2557	7873	2713	1087
2521	1951	9781	1747	9547	1597	9811	1741	1213	9181	9883	1987	9721

Over time, numerous varieties of patterns, including magic squares, were created as a spiritual power. These influences stem from Hermetic geometry, where the illustrations symbolize extraterrestrial shapes. Becoming familiar with these unique shapes and patterns, such as geometric shapes and the Pythagorean Theorem, can help lay a foundation that students will utilize in future studies.

21 x 21 Chart - Number Patterns

The number twenty-one has several significant accolades to its name. It is a Fibonacci number and a Harshad number, which is an integer that is divisible by the sum of its digits when written in that base. It is also the sum of the first six natural numbers, earning the title of a triangular number. Additionally, twenty-one is and an octagonal number, a composite number, with its divisors being 1, 3, and 7 (all prime), and a Motzkin number (Numbermatics, 2016).

Below is a 21 x 21 chart, starting with number 1. Given this information of the quantity and structure (using numbers 1 through 441, since 21 x 21 = 441), there are several apparent patterns throughout this chart each, highlighted in different colors for ease of reading.

21 x 21 Chart – Multiples of 6

1	2	3	4	5	**6**	7	8	9	10	11	**12**	13	14	15	16	17	**18**	19	20	21
22	23	**24**	25	26	27	28	29	**30**	31	32	33	34	35	**36**	37	38	39	40	41	**42**
43	44	45	46	47	**48**	49	50	51	52	53	**54**	55	56	57	58	59	**60**	61	62	63
64	65	**66**	67	68	69	70	71	**72**	73	74	75	76	77	**78**	79	80	81	82	83	**84**
85	86	87	88	89	**90**	91	92	93	94	95	**96**	97	98	99	100	101	**102**	103	104	105
106	107	**108**	109	110	111	112	113	**114**	115	116	117	118	119	**120**	121	122	123	124	125	**126**
127	128	129	130	131	**132**	133	134	135	136	137	**138**	139	140	141	142	143	**144**	145	146	147
148	149	**150**	151	152	153	154	155	**156**	157	158	159	160	161	**162**	163	164	165	166	167	**168**
169	170	171	172	173	**174**	175	176	177	178	179	**180**	181	182	183	184	185	**186**	187	188	189
190	191	**192**	193	194	195	196	197	**198**	199	200	201	202	203	**204**	205	206	207	208	209	**210**
211	212	213	214	215	**216**	217	218	219	220	221	**222**	223	224	225	226	227	**228**	229	230	231
232	233	**234**	235	236	237	238	239	**240**	241	242	243	244	245	**246**	247	248	249	250	251	**252**
253	254	255	256	257	**258**	259	260	261	262	263	**264**	265	266	267	268	269	**270**	271	272	273
274	275	**276**	277	278	279	280	281	**282**	283	284	285	286	287	**288**	289	290	291	292	293	**294**
295	296	297	298	299	**300**	301	302	303	304	305	**306**	307	308	309	310	311	**312**	313	314	315
316	317	**318**	319	320	321	322	323	**324**	325	326	327	328	329	**330**	331	332	333	334	335	**336**
337	338	339	340	341	**342**	343	344	345	346	347	**348**	349	350	351	352	353	**354**	355	356	357
358	359	**360**	361	362	363	364	365	**366**	367	368	369	370	371	**372**	373	374	375	376	377	**378**
379	380	381	382	383	**384**	385	386	387	388	389	**390**	391	392	393	394	395	**396**	397	398	399
400	401	**402**	403	404	405	406	407	**408**	409	410	411	412	413	**414**	415	416	417	418	419	**420**
421	422	423	424	425	**426**	427	428	429	430	431	**432**	433	434	435	436	437	**438**	439	440	441

In the chart above, multiples of 6 are highlighted in pink to show that in every third row, every other number is a multiple of 6.

21 x 21 Chart – Prime Numbers

1	**2**	**3**	4	**5**	6	**7**	8	9	10	**11**	12	**13**	14	15	16	**17**	18	**19**	20	21
22	**23**	24	25	26	27	28	**29**	30	**31**	32	33	34	35	36	**37**	38	39	40	**41**	42
43	44	45	46	**47**	48	49	50	51	52	**53**	54	55	56	57	58	**59**	60	**61**	62	63
64	65	66	**67**	68	69	70	**71**	72	**73**	74	75	76	77	78	**79**	80	81	82	**83**	84
85	86	87	88	**89**	90	91	92	93	94	95	96	**97**	98	99	100	**101**	102	**103**	104	105
106	**107**	108	**109**	110	111	112	**113**	114	115	116	117	118	119	120	121	122	123	124	125	126
127	128	129	130	**131**	132	133	134	135	136	**137**	138	**139**	140	141	142	143	144	145	146	147
148	**149**	150	**151**	152	153	154	155	156	**157**	158	159	160	161	162	**163**	164	165	166	**167**	168
169	170	171	172	**173**	174	175	176	177	178	**179**	180	**181**	182	183	184	185	186	187	188	189
190	**191**	192	**193**	194	195	196	**197**	198	**199**	200	201	202	203	204	205	206	207	208	209	210
211	212	213	214	215	216	217	218	219	220	221	222	**223**	224	225	226	**227**	228	**229**	230	231
232	**233**	234	235	236	237	238	**239**	240	**241**	242	243	244	245	246	247	248	249	250	**251**	252
253	254	255	256	**257**	258	259	260	261	262	**263**	264	265	266	267	268	**269**	270	**271**	272	273
274	275	276	**277**	278	279	280	**281**	282	**283**	284	285	286	287	288	289	290	291	292	**293**	294
295	296	297	298	299	300	301	302	303	304	305	306	**307**	308	309	310	**311**	312	**313**	314	315
316	**317**	318	319	320	321	322	323	324	325	326	327	328	329	330	**331**	332	333	334	335	336
337	338	339	340	341	342	343	344	345	346	**347**	348	**349**	350	351	352	**353**	354	355	356	357
358	**359**	360	361	362	363	364	365	366	**367**	368	369	370	371	372	**373**	374	375	376	377	378
379	380	381	382	**383**	384	385	386	387	388	**389**	390	391	392	393	394	395	396	**397**	398	399
400	**401**	402	403	404	405	406	407	408	**409**	410	411	412	413	414	415	416	417	418	**419**	420
421	422	423	424	425	426	427	428	429	430	**431**	432	**433**	434	435	436	437	438	**439**	440	441

In the chart above, prime numbers are highlighted in maroon. Although it 's challenge to determine a precise pattern for primes, it is interesting to note that at least each column contains, at least, three prime numbers, and of course, 3 is a factor of 21.

Additional patterns for the 21 x 21 number chart can be found below.

21 x 21 Chart – Multiples of 3

1	2	**3**	4	5	**6**	7	8	**9**	10	11	**12**	13	14	**15**	16	17	**18**	19	20	**21**
22	23	**24**	25	26	**27**	28	29	**30**	31	32	**33**	34	35	**36**	37	38	**39**	40	41	**42**
43	44	**45**	46	47	**48**	49	50	**51**	52	53	**54**	55	56	**57**	58	59	**60**	61	62	**63**
64	65	**66**	67	68	**69**	70	71	**72**	73	74	**75**	76	77	**78**	79	80	**81**	82	83	**84**
85	86	**87**	88	89	**90**	91	92	**93**	94	95	**96**	97	98	**99**	100	101	**102**	103	104	**105**
106	107	**108**	109	110	**111**	112	113	**114**	115	116	**117**	118	119	**120**	121	122	**123**	124	125	**126**
127	128	**129**	130	131	**132**	133	134	**135**	136	137	**138**	139	140	**141**	142	143	**144**	145	146	**147**
148	149	**150**	151	152	**153**	154	155	**156**	157	158	**159**	160	161	**162**	163	164	**165**	166	167	**168**
169	170	**171**	172	173	**174**	175	176	**177**	178	179	**180**	181	182	**183**	184	185	**186**	187	188	**189**
190	191	**192**	193	194	**195**	196	197	**198**	199	200	**201**	202	203	**204**	205	206	**207**	208	209	**210**
211	212	**213**	214	215	**216**	217	218	**219**	220	221	**222**	223	224	**225**	226	227	**228**	229	230	**231**
232	233	**234**	235	236	**237**	238	239	**240**	241	242	**243**	244	245	**246**	247	248	**249**	250	251	**252**
253	254	**255**	256	257	**258**	259	260	**261**	262	263	**264**	265	266	**267**	268	269	**270**	271	272	**273**
274	275	**276**	277	278	**279**	280	281	**282**	283	284	**285**	286	287	**288**	289	290	**291**	292	293	**294**
295	296	**297**	298	299	**300**	301	302	**303**	304	305	**306**	307	308	**309**	310	311	**312**	313	314	**315**
316	317	**318**	319	320	**321**	322	323	**324**	325	326	**327**	328	329	**330**	331	332	**333**	334	335	**336**
337	338	**339**	340	341	**342**	343	344	**345**	346	347	**348**	349	350	**351**	352	353	**354**	355	356	**357**
358	359	**360**	361	362	**363**	364	365	**366**	367	368	**369**	370	371	**372**	373	374	**375**	376	377	**378**
379	380	**381**	382	383	**384**	385	386	**387**	388	389	**390**	391	392	**393**	394	395	**396**	397	398	**399**
400	401	**402**	403	404	**405**	406	407	**408**	409	410	**411**	412	413	**414**	415	416	**417**	418	419	**420**
421	422	**423**	424	425	**426**	427	428	**429**	430	431	**432**	433	434	**435**	436	437	**438**	439	440	**441**

21 x 21 Chart – Multiples of 6

1	2	3	**4**	5	6	7	**8**	9	10	11	**12**	13	14	15	**16**	17	18	19	**20**	21
22	23	**24**	25	26	27	**28**	29	30	31	**32**	33	34	35	**36**	37	38	39	**40**	41	42
43	**44**	45	46	47	**48**	49	50	51	**52**	53	54	55	**56**	57	58	59	**60**	61	62	63
64	65	66	67	**68**	69	70	71	**72**	73	74	75	**76**	77	78	79	**80**	81	82	83	**84**
85	86	87	**88**	89	90	91	**92**	93	94	95	**96**	97	98	99	**100**	101	102	103	**104**	105
106	107	**108**	109	110	111	**112**	113	114	115	**116**	117	118	119	**120**	121	122	123	**124**	125	126
127	**128**	129	130	131	**132**	133	134	135	**136**	137	138	139	**140**	141	142	143	**144**	145	146	147
148	149	150	151	**152**	153	154	155	**156**	157	158	159	**160**	161	162	163	**164**	165	166	167	**168**
169	170	171	**172**	173	174	175	**176**	177	178	179	**180**	181	182	183	**184**	185	186	187	**188**	189
190	191	**192**	193	194	195	**196**	197	198	199	**200**	201	202	203	**204**	205	206	207	**208**	209	210
211	**212**	213	214	215	**216**	217	218	219	**220**	221	222	223	**224**	225	226	227	**228**	229	230	231
232	233	234	235	**236**	237	238	239	**240**	241	242	243	**244**	245	246	247	**248**	249	250	251	**252**
253	254	255	**256**	257	258	259	**260**	261	262	263	**264**	265	266	267	**268**	269	270	271	**272**	273
274	275	**276**	277	278	279	**280**	281	282	283	**284**	285	286	287	**288**	289	290	291	**292**	293	294
295	**296**	297	298	299	**300**	301	302	303	**304**	305	306	307	**308**	309	310	311	**312**	313	314	315
316	317	318	319	**320**	321	322	323	**324**	325	326	327	**328**	329	330	331	**332**	333	334	335	**336**
337	338	339	**340**	341	342	343	**344**	345	346	347	**348**	349	350	351	**352**	353	354	355	**356**	357
358	359	**360**	361	362	363	**364**	365	366	367	**368**	369	370	371	**372**	373	374	375	**376**	377	378
379	**380**	381	382	383	**384**	385	386	387	**388**	389	390	391	**392**	393	394	395	**396**	397	398	399
400	401	402	403	**404**	405	406	407	**408**	409	410	411	**412**	413	414	415	**416**	417	418	419	**420**
421	422	423	**424**	425	426	427	**428**	429	430	431	**432**	433	434	435	**436**	437	438	439	**440**	441

21 x 21 Chart – Multiples of 5

1	2	3	4	**5**	6	7	8	9	**10**	11	12	13	14	**15**	16	17	18	19	**20**	21
22	23	24	**25**	26	27	28	29	**30**	31	32	33	34	**35**	36	37	38	39	**40**	41	42
43	44	**45**	46	47	48	49	**50**	51	52	53	54	**55**	56	57	58	59	**60**	61	62	63
64	**65**	66	67	68	69	**70**	71	72	73	74	**75**	76	77	78	79	**80**	81	82	83	84
85	86	87	88	89	**90**	91	92	93	94	**95**	96	97	98	99	**100**	101	102	103	104	**105**
106	107	108	109	**110**	111	112	113	114	**115**	116	117	118	119	**120**	121	122	123	124	**125**	126
127	128	129	**130**	131	132	133	134	**135**	136	137	138	139	**140**	141	142	143	144	**145**	146	147
148	149	**150**	151	152	153	154	**155**	156	157	158	159	**160**	161	162	163	164	**165**	166	167	168
169	**170**	171	172	173	174	**175**	176	177	178	179	**180**	181	182	183	184	**185**	186	187	188	189
190	191	192	193	194	**195**	196	197	198	199	**200**	201	202	203	204	**205**	206	207	208	209	**210**
211	212	213	214	**215**	216	217	218	219	**220**	221	222	223	224	**225**	226	227	228	229	**230**	231
232	233	234	**235**	236	237	238	239	**240**	241	242	243	244	**245**	246	247	248	249	**250**	251	252
253	254	**255**	256	257	258	259	**260**	261	262	263	264	**265**	266	267	268	269	**270**	271	272	273
274	**275**	276	277	278	279	**280**	281	282	283	284	**285**	286	287	288	289	**290**	291	292	293	294
295	296	297	298	299	**300**	301	302	303	304	**305**	306	307	308	309	**310**	311	312	313	314	**315**
316	317	318	319	**320**	321	322	323	324	**325**	326	327	328	329	**330**	331	332	333	334	**335**	336
337	338	339	**340**	341	342	343	344	**345**	346	347	348	349	**350**	351	352	353	354	**355**	356	357
358	359	**360**	361	362	363	364	**365**	366	367	368	369	**370**	371	372	373	374	**375**	376	377	378
379	**380**	381	382	383	384	**385**	386	387	388	389	**390**	391	392	393	394	**395**	396	397	398	399
400	401	402	403	404	**405**	406	407	408	409	**410**	411	412	413	414	**415**	416	417	418	419	**420**
421	422	423	424	**425**	426	427	428	429	**430**	431	432	433	434	**435**	436	437	438	439	**440**	441

21 x 21 Chart – Multiples of 9

1	2	3	4	5	6	7	8	**9**	10	11	12	13	14	15	16	17	**18**	19	20	21
22	23	24	25	26	**27**	28	29	30	31	32	33	34	35	**36**	37	38	39	40	41	42
43	44	**45**	46	47	48	49	50	51	52	53	**54**	55	56	57	58	59	60	61	62	**63**
64	65	66	67	68	69	70	71	**72**	73	74	75	76	77	78	79	80	**81**	82	83	84
85	86	87	88	89	**90**	91	92	93	94	95	96	97	98	**99**	100	101	102	103	104	105
106	107	**108**	109	110	111	112	113	114	115	116	**117**	118	119	120	121	122	123	124	125	**126**
127	128	129	130	131	132	133	134	**135**	136	137	138	139	140	141	142	143	**144**	145	146	147
148	149	150	151	152	**153**	154	155	156	157	158	159	160	161	**162**	163	164	165	166	167	168
169	170	**171**	172	173	174	175	176	177	178	179	**180**	181	182	183	184	185	186	187	188	**189**
190	191	192	193	194	195	196	197	**198**	199	200	201	202	203	204	205	206	**207**	208	209	210
211	212	213	214	215	**216**	217	218	219	220	221	222	223	224	**225**	226	227	228	229	230	231
232	233	**234**	235	236	237	238	239	240	241	242	**243**	244	245	246	247	248	249	250	251	**252**
253	254	255	256	257	258	259	260	**261**	262	263	264	265	266	267	268	269	**270**	271	272	273
274	275	276	277	278	**279**	280	281	282	283	284	285	286	287	**288**	289	290	291	292	293	294
295	296	**297**	298	299	300	301	302	303	304	305	**306**	307	308	309	310	311	312	313	314	**315**
316	317	318	319	320	321	322	323	**324**	325	326	327	328	329	330	331	332	**333**	334	335	336
337	338	339	340	341	**342**	343	344	345	346	347	348	349	350	**351**	352	353	354	355	356	357
358	359	**360**	361	362	363	364	365	366	367	368	**369**	370	371	372	373	374	375	376	377	**378**
379	380	381	382	383	384	385	386	**387**	388	389	390	391	392	393	394	395	**396**	397	398	399
400	401	402	403	404	**405**	406	407	408	409	410	411	412	413	**414**	415	416	417	418	419	420
421	422	**423**	424	425	426	427	428	429	430	431	**432**	433	434	435	436	437	438	439	440	**441**

21 x 21 Chart - Magic Square

233	256	279	302	325	348	371	394	417	440	1	24	47	70	93	116	139	162	185	208	231
255	278	301	324	347	370	393	416	439	21	23	46	69	92	115	138	161	184	207	230	232
277	300	323	346	369	392	415	438	20	22	45	68	91	114	137	160	183	206	229	252	254
299	322	345	368	391	414	437	19	42	44	67	90	113	136	159	182	205	228	251	253	276
321	344	367	390	413	436	18	41	43	66	89	112	135	158	181	204	227	250	273	275	298
343	366	389	412	435	17	40	63	65	88	111	134	157	180	203	226	249	272	274	297	320
365	388	411	434	16	39	62	64	87	110	133	156	179	202	225	248	271	294	296	319	342
387	410	433	15	38	61	84	86	109	132	155	178	201	224	247	270	293	295	318	341	364
409	432	14	37	60	83	85	108	131	154	177	200	223	246	269	292	315	317	340	363	386
431	13	36	59	82	105	107	130	153	176	199	222	245	268	291	314	316	339	362	385	408
12	35	58	81	104	106	129	152	175	198	221	244	267	290	313	336	338	361	384	407	430
34	57	80	103	126	128	151	174	197	220	243	266	289	312	335	337	360	383	406	429	11
56	79	102	125	127	150	173	196	219	242	265	288	311	334	357	359	382	405	428	10	33
78	101	124	147	149	172	195	218	241	264	287	310	333	356	358	381	404	427	9	32	55
100	123	146	148	171	194	217	240	263	286	309	332	355	378	380	403	426	8	31	54	77
122	145	168	170	193	216	239	262	285	308	331	354	377	379	402	425	7	30	53	76	99
144	167	169	192	215	238	261	284	307	330	353	376	399	401	424	6	29	52	75	98	121
166	189	191	214	237	260	283	306	329	352	375	398	400	423	5	28	51	74	97	120	143
188	190	213	236	259	282	305	328	351	374	397	420	422	4	27	50	73	96	119	142	165
210	212	235	258	281	304	327	350	373	396	419	421	3	26	49	72	95	118	141	164	187
211	234	257	280	303	326	349	372	395	418	441	2	25	48	71	94	117	140	163	186	209

The figure above is a 21 x 21 magic square. Each of the rows, columns, and diagonals will add to 4,641. A magic square can be found by either adding each of the rows, columns, and diagonals, or by using the formula $(n(n^2 + 1) / 2)$. To create this magic square, the number 1 is placed in the middle of the upper column (highlighted baby blue), numbers are then "wrapped around" the square vertically and horizontally. If you look closely, you will see that the numbers 1-21 are highlighted in baby blue, following a diagonal pattern. Numbers 22-42, highlighted in purple, also follow this pattern. In fact, every 21 numbers (43-63, 64-84, 85-105, etc.) will follow this exact pattern, creating a truly impressive puzzle.

37 x 37 Chart - Number Patterns

The pattern of 37 will fill out with 1369 boxes since it's consists of 37 by 37 or n by n table. One of the advantages of analyzing number patterns is that often it is possible to see a

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74
75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111
112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148
149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185
186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222
223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259
260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296
297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327	328	329	330	331	332	333
334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363	364	365	366	367	368	369	370
371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401	402	403	404	405	406	407
408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441	442	443	444
445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	460	461	462	463	464	465	466	467	468	469	470	471	472	473	474	475	476	477	478	479	480	481
482	483	484	485	486	487	488	489	490	491	492	493	494	495	496	497	498	499	500	501	502	503	504	505	506	507	508	509	510	511	512	513	514	515	516	517	518
519	520	521	522	523	524	525	526	527	528	529	530	531	532	533	534	535	536	537	538	539	540	541	542	543	544	545	546	547	548	549	550	551	552	553	554	555
556	557	558	559	560	561	562	563	564	565	566	567	568	569	570	571	572	573	574	575	576	577	578	579	580	581	582	583	584	585	586	587	588	589	590	591	592
593	594	595	596	597	598	599	600	601	602	603	604	605	606	607	608	609	610	611	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629
630	631	632	633	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648	649	650	651	652	653	654	655	656	657	658	659	660	661	662	663	664	665	666
667	668	669	670	671	672	673	674	675	676	677	678	679	680	681	682	683	684	685	686	687	688	689	690	691	692	693	694	695	696	697	698	699	700	701	702	703
704	705	706	707	708	709	710	711	712	713	714	715	716	717	718	719	720	721	722	723	724	725	726	727	728	729	730	731	732	733	734	735	736	737	738	739	740
741	742	743	744	745	746	747	748	749	750	751	752	753	754	755	756	757	758	759	760	761	762	763	764	765	766	767	768	769	770	771	772	773	774	775	776	777
778	779	780	781	782	783	784	785	786	787	788	789	790	791	792	793	794	795	796	797	798	799	800	801	802	803	804	805	806	807	808	809	810	811	812	813	814
815	816	817	818	819	820	821	822	823	824	825	826	827	828	829	830	831	832	833	834	835	836	837	838	839	840	841	842	843	844	845	846	847	848	849	850	851
852	853	854	855	856	857	858	859	860	861	862	863	864	865	866	867	868	869	870	871	872	873	874	875	876	877	878	879	880	881	882	883	884	885	886	887	888
889	890	891	892	893	894	895	896	897	898	899	900	901	902	903	904	905	906	907	908	909	910	911	912	913	914	915	916	917	918	919	920	921	922	923	924	925
926	927	928	929	930	931	932	933	934	935	936	937	938	939	940	941	942	943	944	945	946	947	948	949	950	951	952	953	954	955	956	957	958	959	960	961	962
963	964	965	966	967	968	969	970	971	972	973	974	975	976	977	978	979	980	981	982	983	984	985	986	987	988	989	990	991	992	993	994	995	996	997	998	999
1000	1001	1002	1003	1004	1005	1006	1007	1008	1009	1010	1011	1012	1013	1014	1015	1016	1017	1018	1019	1020	1021	1022	1023	1024	1025	1026	1027	1028	1029	1030	1031	1032	1033	1034	1035	1036
1037	1038	1039	1040	1041	1042	1043	1044	1045	1046	1047	1048	1049	1050	1051	1052	1053	1054	1055	1056	1057	1058	1059	1060	1061	1062	1063	1064	1065	1066	1067	1068	1069	1070	1071	1072	1073
1074	1075	1076	1077	1078	1079	1080	1081	1082	1083	1084	1085	1086	1087	1088	1089	1090	1091	1092	1093	1094	1095	1096	1097	1098	1099	1100	1101	1102	1103	1104	1105	1106	1107	1108	1109	1110
1111	1112	1113	1114	1115	1116	1117	1118	1119	1120	1121	1122	1123	1124	1125	1126	1127	1128	1129	1130	1131	1132	1133	1134	1135	1136	1137	1138	1139	1140	1141	1142	1143	1144	1145	1146	1147
1148	1149	1150	1151	1152	1153	1154	1155	1156	1157	1158	1159	1160	1161	1162	1163	1164	1165	1166	1167	1168	1169	1170	1171	1172	1173	1174	1175	1176	1177	1178	1179	1180	1181	1182	1183	1184
1185	1186	1187	1188	1189	1190	1191	1192	1193	1194	1195	1196	1197	1198	1199	1200	1201	1202	1203	1204	1205	1206	1207	1208	1209	1210	1211	1212	1213	1214	1215	1216	1217	1218	1219	1220	1221
1222	1223	1224	1225	1226	1227	1228	1229	1230	1231	1232	1233	1234	1235	1236	1237	1238	1239	1240	1241	1242	1243	1244	1245	1246	1247	1248	1249	1250	1251	1252	1253	1254	1255	1256	1257	1258
1259	1260	1261	1262	1263	1264	1265	1266	1267	1268	1269	1270	1271	1272	1273	1274	1275	1276	1277	1278	1279	1280	1281	1282	1283	1284	1285	1286	1287	1288	1289	1290	1291	1292	1293	1294	1295
1296	1297	1298	1299	1300	1301	1302	1303	1304	1305	1306	1307	1308	1309	1310	1311	1312	1313	1314	1315	1316	1317	1318	1319	1320	1321	1322	1323	1324	1325	1326	1327	1328	1329	1330	1331	1332
1333	1334	1335	1336	1337	1338	1339	1340	1341	1342	1343	1344	1345	1346	1347	1348	1349	1350	1351	1352	1353	1354	1355	1356	1357	1358	1359	1360	1361	1362	1363	1364	1365	1366	1367	1368	1369

small part of it.

There are few easy patterns to find out on the 37 by 37 table. The first one is that the right diagonal is increased by 36 from 1 until 1369. For instance, (37 + 36 = 73, 73 + 36 = 109, 1297 + 36 = 1333). On the other hand, the left diagonal is augmented by 38. For example, (1 + 38 = 39, 39 + 38 = 77, 1331 + 38 = 1369). Also, it is possible to notice that subtracting any number starting from the second row from the previous one the result will always be 37. For instance, (38 – 1 = 37, 704 – 667 = 37, 1060 – 1023 = 37, 962 – 925 = 37, 1369 – 1332 = 37). Also, the third row is equal to the multiplication of the second row minus the first row. Row3 = R2 x 2 – R1: (79 = 42 x 2 – 5, 99 = 62 x 2 – 25, 111 = 74 x 2 – 37)

According to the author Chris K. Caldwell (2015), there are plenty conjectures related to prime number. One of them says that every even n > 2 is the sum of two primes which comes from the mathematician Goldbach's work. Back in 1742, Goldback sends a mathematical proof in a letter to Euler suggesting that every integer n > 5 is equal to the sum of three prime's numbers. After Euler analyzing his friend proposal, he found out that every even number greater than two is equal to the sum of two prime numbers. Also, another important prime conjecture is more familiar. In fact, there are infinitely many twin primes such as "the sum of the reciprocals of the twin primes converges, as so the sum B = (1/3 + 1/5) + (1/5 + 1/7) + (1/11 + 1/13) + (1/17 + 1/19) + is Brun's constant."

$$\boxed{2\prod_{p\geq 3}\frac{p(p-2)}{(p-1)^2}\int_2^x\frac{dx}{(\log x)^2} \doteq 1.320323632\int_2^x\frac{dx}{(\log x)^2}}$$

The conjecture formula above proved that there are about twin primes less than or equal to x, and it also shows that there is an infinite product of the twin primes constants (Caldwell, C. K., 2015).

The following activity created by “The Hong Kong Academy for Gifted Students,” can be used to teach the conceptions of prime numbers along with the conjectures of multiples of six.

Present the statement to pupils in either of the following ways:

“All prime numbers greater than three can be expressed in the form 6n + 1, 6n – 1, where n is a positive whole number.”
“Every prime number greater than 3 is either one more than or one more than a multiple of 6.”

Give the students some time to investigate the above statement (preferably using Excel) and encourage them to come up with a proof of this proposition.

Investigation in Excel:

n	6n - 2	**6n - 1**	6n	**6n + 1**	6n + 2	6n + 3
0				1	2	3
1	4	5	6	7	8	9
2	10	11	12	13	14	15
3	16	17	18	19	20	21
4	22	23	24	25	26	27
5	28	29	30	31	32	33
6	34	35	36	37	38	39
7	40	41	42	43	44	45
8	46	47	48	49	50	51
9	52	53	54	55	56	57
10	58	59	60	61	62	63
11	64	65	66	67	68	69
12	70	71	72	73	74	75

The results show that:

(i) All prime numbers so far tested (apart from a and 3) lie either in the 6n + 1 column or the 6n – 1 column. There are no primes in the other columns.

(ii) There are, however, non-primes in the 6n + 1 column and the 6n – 1 column.

So far all we can see is that some examples fit the conjecture. We have not yet proved that all primes fit the conjecture. It would take forever!

Suggested proof:

Star by noticing that every whole number can be expressed in the form 6n – 2, 6n -1, 6n +1, 6n +2, and 6n + 3. Then, notice the following facts:

(i) 6n is always divisible by 6, for all values of n (so none of the numbers in this column can be prime).

(ii) 6n – 2 is always divisible by 2, for all values of n (so none of the numbers in this column can be prime either, except two itself).

(iii) The same is true of 6n + 2.

(iv) 6n + 3 is always divisible by 3, for all values of n (so none of the numbers in this column can be prime, except three itself). So, all the primes greater than three must lie in the 6n – 1 and 6n + 1 columns. (The Hong Kong Academy for Gifted Students, 2003).

University of Tennessee-Martin Professor Chris Caldwell wrote that he (along with a friend using MATLAB (computer programming language)) found every prime number over three lies next to a number divisible by six. After testing 1,000,000 prime numbers, take $n > 3$ and divide it by $6n = 6q + r$ where q is a non-negative integer and the remainder r is one of 0, 1, 2, 3, 4, or 5.

- If the remainder is 0, 2 or 4, then the number n is divisible by 2, and cannot be prime.
- If the remainder is 3, then the number n is divisible by 3, and cannot be prime.

So if n is prime, then the remainder r is either

- 1 (and $n = 6q + 1$ is one more than a multiple of six), or
- 5 (and $n = 6q + 5 = 6(q+1) - 1$ is one less than a multiple of six).

Remember that being one more or less than a multiple of six does not make a number prime. We have only shown that all primes other than 2 and 3 (which divide 6) have this form.

Classroom Uses for Number Patterns

The hundreds chart can be used in a variety of ways in a math class. As a result, we have noticed that the other number charts are not better for teaching students of all age simple but complex math pattern. Furthermore, a hundreds chart helps students see patterns with numbers. The hundreds chart can also be used to help students with many number sense related activities, as compared to each of the four mathematical operations of addition, subtraction, multiplication, and division. In the 10x10 hundred chart, the numbers 1-100 are arranged in ten columns and ten rows. Within the columns and rows of the 10 x 10 hundred chart, there are several patterns in the chart, which can be identified. In looking at a 10 x 10 hundred chart, many number patterns that are evident. Not only are there many patterns that are evident in the 10 x 10 hundred chart, but there is also a basic pattern (a formula) for the basic 10 x 10 hundred charts (square). The number patterns that are present in a 10 x 10 hundred chart lend themselves to a learning tool for students of all ages.

Magic squares are proving to be an ideal tool for the effective illustration of many mathematical concepts. In fact, simple Arithmetic, which would stem from summing the numbers in the rows, columns, and diagonals, to Algebra and Geometry with the application of the formula mentioned in the first section of this paper (Anderson) here are many ways to incorporate magic squares to help teach students math. Consider a few important things when adapting magic squares for the classroom.

It is imperative to make sure the students understand a method of determining the placement of numerals in any size magic cell – this can be done by providing students with an example first – perhaps a 3 x 3 or 4 x 4 magic square. When providing the example, point out how the numbers in the squares add to the same number in every direction. To make this more concrete to the student, the instructor could provide a second example that uses the same square, but with one or more cells missing. Students could then either find the missing consecutive number (arithmetic) or create a formula to find the missing number that would make the "magic" happen. For an illustration of this, see the example below.

8	**1**	**6**
3	**5**	**7**
4	**9**	**2**

Note: Introduce the lesson by displaying a full 3 x 3 magic square – point out the sum of the rows, columns, and diagonals is 15. Also introduce the formula $M = n(n^2 + 1)/2$ to show that the magic number is 15.

16	**2**	
6	**10**	
	18	**4**

Note: Use progression by starting to block out the cells. You can use the same number, or double or triple the number. In this example, the numbers are doubled. Ask students to fill in the missing numbers. They should be able to point out that M is 30, and proceed to fill in the missing numbers from there. Circulate and provide guidance if necessary.

After completing this exercise with students, lead students to develop an understanding of a method of constructing a magic square by attempting to create one of their own. Some examples of exercises for the classroom are listed:

Exercise 1: Draw a 3 by 3 grid, and without any clues, see if the students can fill in the numbers 1 to 9 so that the result is a magic square. If you want to give a hint; put the number 5 in the middle.

Exercise 2: Look at your final result from the last magic square; now have the students square every number in the square. Is the square still magic? Yes! Be sure to ask students why to ensure they understand this. Use a simple formula, like x + 3 = 8 – students will say that x = 5. Now double the numbers – 2x + 6 = 16 – students will still say that x = 5.. Finally, show them the formula $M = n(n^2 + 1)/2$ and use any number for n, then ask them to square n and complete the formula again. The result should be M^2!

Exercise 3: To begin, refer back to the first example, the 3 x 3 magic square. In this square, the middle number is 5 and is in the center of the square. Ask students to try putting another number in the center. After some time, it will be discovered that no other number will work in the center position. Therefore, it can be concluded that because 5 is the median number, it must be placed in the center position with a greater and lesser number on either side.
For this exercise, provide the students with nine prime numbers: 1, 7, 13, 31, 37, 43, 61, 67, and 73. Can these numbers be arranged into a magic square? Be sure to remind students the key to arranging the numbers correctly in any magic square is to realize that the middle number (in this case, 37) must always go in the center.

67	1	43
13	37	61
31	73	7

Exercise 4: Another activity could be a 3 x 3 Magic Square: write all the number 1-9 on small squares of paper and cut them out; move the numbers to the spaces so the sum of each row, column, and main diagonal equals 15 and have the students record their work. You can challenge them by asking if there is more than one way to place the numbers to that the sums of each row, column, and main diagonal equals 15—have them compare with other students!

Exercise 5: A 5 x 5 Magic Square: write all the number 1-25 on small squares of paper and cut them out; move the numbers to the spaces so the sum of each row, column, and main diagonal equals 65 and have the students record their work.

The key to the mastery of this concept is building a solid foundation for students to work.

By demonstrating the mathematical concept you are trying to get students to comprehend in an artless manner first, you are creating a bridge into the understanding of more complicated mathematical concepts, like the magic square. Starting small and taking the time to ask why, explain concepts, and demonstrate why the square is magic will allow you and your students to grasp fully the magical properties of this phenomenal concept (Anderson).

Closing Remarks

Numerical patterns are just the beginning of the acknowledgement of the importance of mathematics in one's everyday life. Through careful observation and conjecture we have found that numerous patterns in both number charts and magic squares. These observations can be passed along to students beginning at an early age – both allowing them to deepen their knowledge of the number system, and develop an awareness for patterns and puzzles in the study of mathematics. It is our hope that the teaching of numerical patterns to elementary age children will also develop a love of the beauty and presence of mathematics in our everyday lives.

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