Physics Lab Experiments with Simulated Data for Remote Delivery

Department of Chemistry and Physics

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Experiment 2.01: Coulomb's Law

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I. EXPERIMENT 2.01: COULOMB'S LAW

A. Abstract

The attractive force between a plane charge (in the form of a triboelectrically charged Teflon plate) and an uncharged conducting sphere is investigated. The charge induction on the sphere results in a separation of positive from negative charge on its surface and consequently an electric dipole moment.

B. Formulas

The mathematical statement of Coulomb's law for point charges is

$$F = k_e \frac{|q_1 q_2|}{r^2} \tag{1}$$

$$k_e = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \ N \cdot m^2/C^2 \tag{2}$$

where $q_{1,2}$ are the electric charges (in Coulomb units) associated with two point particles. This is basis of the the equations that follow.

C. Description and Background

In the experiment, a Teflon plate (of area $A = L \times W$) will acquire a charge, Q, from frictional contact with a wool felt cloth. We will assume the charge is uniformly distributed on the plate's surface, so the surface charge density is

$$\sigma = \frac{Q}{A} \tag{3}$$

When the plate is placed near an uncharged conducting sphere (of radius R), the two will attract.

Application of Coulomb's law (beyond the scope of this course) to such an arrangement (the sphere equidistant from the four corners of the plate) predicts an attractive force that varies with the distance from the center of the plate to the center of the sphere, r, as follows

$$F_z = \frac{16k\sigma^2 R^3}{\lambda_p} f(r) \tag{4}$$

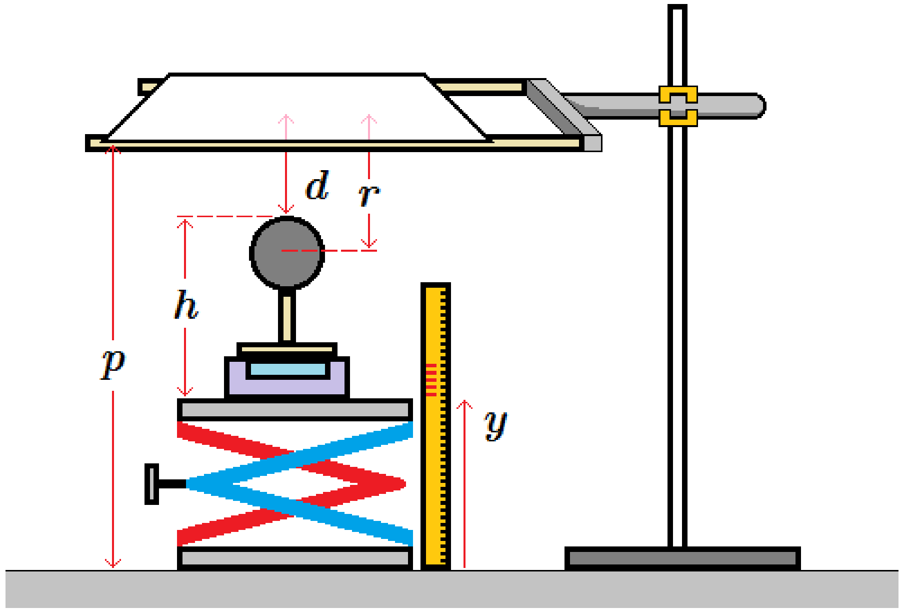

FIG. 1: Experimental arrangement.

where

$$f(r) = \frac{\left(1 + 32\frac{\lambda_p^4}{A^2}\eta^2\right)}{\left(1 + 16\frac{\lambda_p^4}{A^2}\eta^2\right)^{1/2}\left[1 + \eta^2 + 16\frac{\lambda_p^4}{A^2}\eta^4\right]} \sin^{-1}\left(\left[1 + \eta^2 + 16\frac{\lambda_p^4}{A^2}\eta^4\right]^{-1/2}\right)$$

and where $\eta = r/\lambda_p$, and λ_p is a characteristic length for the plate defined by

$$\frac{1}{\lambda_p^2} = \frac{1}{(L/2)^2} + \frac{1}{(W/2)^2} \tag{5}$$

The attractive force (see the diagram for the experimental arrangement) will result in a weight deficit, Δmg, for the sphere that is located on a scale calibrated to zero before the plate is introduced.

Charge is generated on the Teflon plate triboelectrically, *i.e.*, by rubbing with a wool felt cloth in this case. This process of charging the plate is haphazard. Therefore generating a precise charge is unfeasible and precisely reproducing a previously generated charge is also. One of the goals of the experiment is to determine the charge initially generated on the plate. Once charged the Teflon plate will leak its charge to an extent that depends significantly on the environmental humidity. This leakage will be described as an exponential decay

$$Q(t) = Q_0 e^{-\alpha t/2} \tag{6}$$

where $\tau = 2/\alpha$ is the e-folding time, and Q_0 is the charge on the plate at $t = 0$. The goal when charging the plate will be to generate as great a charge as possible. After setting the plate into place, data collection will follow. Each data point will consist of three measurements: (a time, a distance, a mass scale reading). From the equations above, the relation among these variables and parameters can be expressed as follows (taking into account that the attractive force registers a weight deficit, Δmg)

$$\frac{\Delta m}{m_s} = \frac{16kQ_0^2R^3}{m_s g \lambda_p A^2} f(r)\, e^{-\alpha t} \tag{7}$$

where m_s is the mass of the sphere. Taking the natural logarithm of this equation yields

$$\ln\left\{ f(r) \frac{m_s}{\Delta m} \right\} = \alpha t + \gamma \tag{8}$$

where

$$\gamma = \ln\left[\frac{m_s g \lambda_p A^2}{16kQ_0^2R^3} \right] \tag{9}$$

Using the data collected, a linear regression can be performed to arrive at α and γ. From γ, a determination of the initial charge load on the plate is possible. Finally, $2/\alpha$ provides a characteristic time for charge leakage off the plate given the environmental conditions in the laboratory during the experiment.

D. Procedure

1. Set the scale on the lab jack and the conductive sphere on the scale.

2. Adjust the plate support on the retort stand, and place the Teflon plate on it. Make sure that it is horizontal.

3. Arrange it so the midpoint of the Teflon plate is located directly above the sphere. The distance between the top of the sphere and the bottom of the plate is d. By noting the location of the top of the sphere from the base of the lab jack, h, and the height of the bottom of the plate from the table, p, one can easily determine the surface-to-surface distance, $d = p - y - h$. Set the initial surface-to-surface separation of the plate and sphere at $d = 1\ cm$.

4. Remove the Teflon plate and use the wool felt cloth to generate a static charge on the plate by rubbing it with the cloth.

5. Before replacing the plate on the support, turn on the scale and make sure it reads zero. Place the plate on the support. Once the plate is in place, wait for the scale to reach its maximum value and begin to decrease. Start the timer when the value begins to drop. Record the absolute value of the scale reading at 10 *seconds*, then slide the stand with the sphere completely out from under the plate. If the scale returns to zero, slide the stand back in place under the plate; if it does not, zero the scale manually using the TARE button, and return the stand back under the plate. Note the scale reading at 30 *seconds* elapsed time, then at 50 *sec*, 70 *sec*, 90 *sec*, 110 *sec*, 130 *sec*, and 150 *sec*, sliding the stand out between each measurement to ensure a return to zero. Conduct more sets of trials with $d = 2\ cm$, $3\ cm$, ... The readings for Δm will be negative indicating that the force is attractive but record their absolute value.

E. Measurements

base to sphere top, h [cm]	
table to plate, p [cm]	
relative humidity, $R.H.$ [%]	

Note from Figure 1 above that $d = p - y - h$

plate length, L [cm]	
plate width, W [cm]	
sphere radius, R [cm]	
mass of sphere, m_s [$grams$]	

Case 1: $d =$	cm
time, t [sec]	$\|\Delta m\|$ [$grams$]
10	
30	
50	
70	
90	
110	
130	
150	

Case 2: $d =$	cm
time, t [sec]	$\|\Delta m\|$ [$grams$]
10	
30	
50	
70	
90	
110	
130	
150	

F. Instructions

1. Complete the Plotting Tables for each case.Use the link below to the Excel file to perform the calculations and, plot $\ln\left\{f\left(r\right)\frac{m_s}{\Delta m}\right\}$ *vs.* t, and extract the linear regression parameters (α and γ): CoulombsLawLabCalculations.xlsx

2. From Eq. (9)

$$Q_0 = \sqrt{\frac{m_s g \lambda_p A^2}{16 k R^3 e^{\gamma}}} \tag{10}$$

   Use this equation to find the initial charge (in *nano-Coulombs*) on the plates.

3. Based on the charge, Q_0, determined in the experiment, how many net electrons/protons, N, does this charge correspond to?

G. Calculations

A [cm^2]	
λ_p [cm]	

Case 1	
$r = d + R$ [cm]	
r/λ_p	
$f(r)$	
t [sec]	$\ln\{f(r)\, m_s/\lvert\Delta m\rvert\}$
10	
30	
50	
70	
90	
110	
130	
150	

Case 2	
$r = d + R$ [cm]	
r/λ_p	
$f(r)$	
t [sec]	$\ln\{f(r)\, m_s/\lvert\Delta m\rvert\}$
10	
30	
50	
70	
90	
110	
130	
150	

case	α [sec^{-1}]	γ	$\tau = 2/\alpha$ [sec]	Q_0 [nC]	$N = Q_0/e$
1					
2					