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Function Concept: Learning from History

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The importance of functions in school mathematics has grown tremendously within the past century. Functions have progressed from being scantily represented in school mathematics to being a core mathematical topic. C.B. Boyer (1946) acknowledged “The development of the function concept has revolutionized mathematics in much the same way as did the nearly simultaneous rise of non-Euclidean geometry. It has transformed mathematics from a pure natural science- the queen of the sciences- into something vastly large. It has established mathematics as the basis of all rigorous thinking – the logic of all possible relations” (Markovits, Eylor, & Bruckheimer, 1986, p. 18).

Historical speeches and documents, such as Klein’s 1893 Evanston Colloquium, Moore’s 1902 presidential address to the American Mathematical Society, The Reorganization of Mathematics in Secondary Education Report (1923), and The Report of Progressive Education and Joint Committee (1940), advocated that functions and “relational thinking” be a core concept in school mathematics. In fact, Felix Klein considered functions to be the “soul of mathematics”, and advocated that teachers teach functional concepts.

Fortunately, the recommendations made decades ago pertaining to the importance of functions, and the needs to readily integrate the function concept into school mathematics by researchers were not ignored. The recommendations made regarding functions decades ago are evident in today’s curriculum standards. Standards for mathematics require students to be able to define functions, describe functions, identify functions, analyze functions, and recognize patterns in function (NCTM, 2000; Common Core State Standards 2010). Most notably, The Common Core State Standards (2010) has functions as one of five conceptual categories in high school mathematics.

Considering the increased emphasis placed on functions in school mathematics within the past century, we sought to describe how the function concept was presented in secondary mathematics textbooks prior to the “New Math” era.

HISTORICAL OVERVIEW

The function concept has evolved significantly over time: from implicit tools in mathematics and science with no word and no definition to a conceptual category in the Common Core State Standards. Up to the Middle Ages, the concept of function did not

appear with a definition, although, the notion of functional relation existed. During ancient times, tables of squares, tables of square roots, tables of cubics, and table of cubic roots were representations of functions that were visible; as well as geometric figures (Kleiner, 1989; Ponte, 1992). A particular instance of functions in ancient time was the counting of objects; the counting of objects suggests a correspondence between a set of given objects and a sequence of counting numbers (Ponte, 1992).

Felix Klein, a German mathematician, was an advocate for the concept of function being included in school mathematics. Klein accentuated the view that “‘functional thinking’ should be made the binding or unifying principle of school mathematics” (Hamley, 1934a, p.169).

The importance of the function concept was emphasized holistically for secondary mathematics curriculum, in historical mathematics education reports (such as National Committee on Mathematical Requirements 1923 report, and Progressive Education Association 1940 report). Hamley (1934b) noted that the world is becoming “functionally minded”, since functions was increasingly visible in economics, politics, industry and commerce.

Considering the petitions to include functions in school mathematics at the dawn of the twentieth century, our study explored the inclusion of functions in secondary mathematics textbooks during the period 1908-1950, because textbooks were primary resource materials for the teaching of mathematics.

DESIGN OF THE STUDY

We conducted a descriptive study of function concept in school mathematics textbooks. We examined how functions were presented in secondary mathematics textbooks during 1908-1950 in the United States.

We perused the University of Missouri Mathematics Education historical textbooks collection to identify the year functions first appeared. Our initial exploration revealed that the first time the word “functions” was cited, as an independent topic was 1908, hence it was agreed upon that 1908 would be the beginning of the period studied. We restricted the time period to 1950 because it was the last decade before the dawn of the “New Math”, during which more rigid function concept were integrated into school mathematics.

After establishing a time period, we utilized the following criteria to select textbooks: The textbooks must be suitable for high school general mathematics, practical mathematics, or algebra courses; and explicitly reference the word “function” or “functionality”. We collected data from the University of Missouri Mathematics Education historical textbooks collection and from the MOBIUS database. We did not include reprinted books, but kept books that were reprinted with corrections since we were uncertain where the corrections were made. Notwithstanding that 61 of the textbooks examined had a representation of the concept of function, only 36 textbooks provided an explicit definition. Hence for this report, our analysis focuses on 36 of the textbooks in our sample.

Table 1 The number of books in our sample (N =61) that defined functions.

Year of Publication	Explicit Definition of Function	No Explicit Definition of Function
1908-1920	12	13
1921-1930	6	3
1931-1940	8	6
1941-1950	10	3
Total	36	25

We coded data from each textbook for definitions, representations, and importance of functions. We considered language used to define functions (such as vary, correspond, relate, etc). Additionally, having read the definitions of functions we grouped them into three groups based on leading terms (variables, expressions, and numbers/quantities). Subsequently we analyzed our results descriptively.

RESULTS

We present our results under three major themes: definitions of functions, representations of functions, and the importance of functions.

Definitions

There existed differences in the definitions for the verbs used by textbooks to describe the functional relationships between a dependent and independent variable. The verbs used included depends, relates, corresponds, varies, connects, etc. Additionally, we found that 44% of the definitions described functions in terms of variables, 31% described functions in terms of an expression, and the remaining 25% of the definitions described functions as a quantity/number.

Surprisingly, some definitions in secondary school mathematics textbooks presented the possibility of multiple independent variables, while others provided multivalued functions. Two unique definitions were Brooke and Wilcox (1938) (classified in variables), and Ferrar (1948) (classified in expressions). Brooke and Wilcox considered multivalued functions, and Ferrar (1948) defined both implicit and explicit functions. The respective definitions are as follows:

(I) The variable y is said to be a single-valued function of the variable x for a prescribed range of values which the variable x may assume, when a definite value of y corresponds to each value of x , no matter in what manner the correspondence is specified. (II) The variable y is said to be a multiple-valued function of the variable x for a prescribed range of values, which the variable x may assume, when a definite set of values of y corresponds to each value of x no matter in what manner the correspondence is specified (Brooke & Wilcox, 1938, p. 220).

A function like $y = e^x \sin x$ is sometimes called an EXPLICIT FUNCTION, since the definition of y in terms of x is given directly and explicitly. A function y defined in terms of x by means of an equation that fails to give y

directly in terms of x , such as $1 + xe^y = y$, is sometimes called an implicit function, since the equation implies that y depends on x but does not directly and explicitly define y in terms of x (Ferrar, 1948, p. 51).

Table 2 depicts the percentages of the number of independent variables per dependent variables.

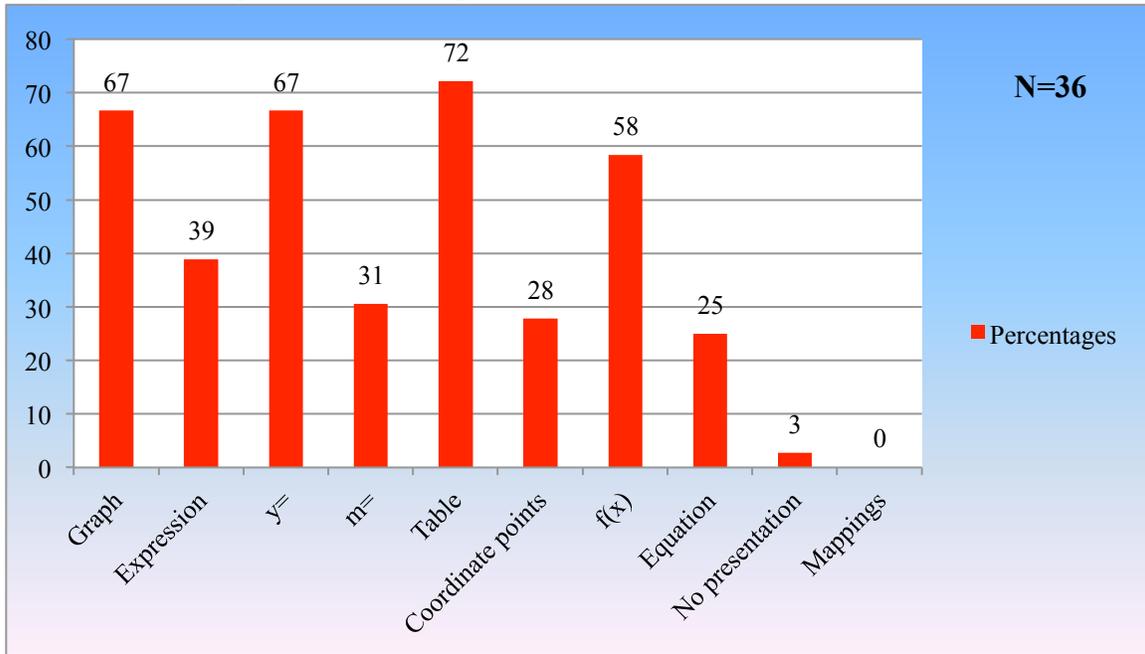
Table 2. The percentages of the number of independent variables in the definitions of functions

Number of Independent Variable(s)	Percentages (N =36)
One variable function only	83%
One variable or multi variables function	11%
Undetermined	6%
Total	100%

Representation of Functions

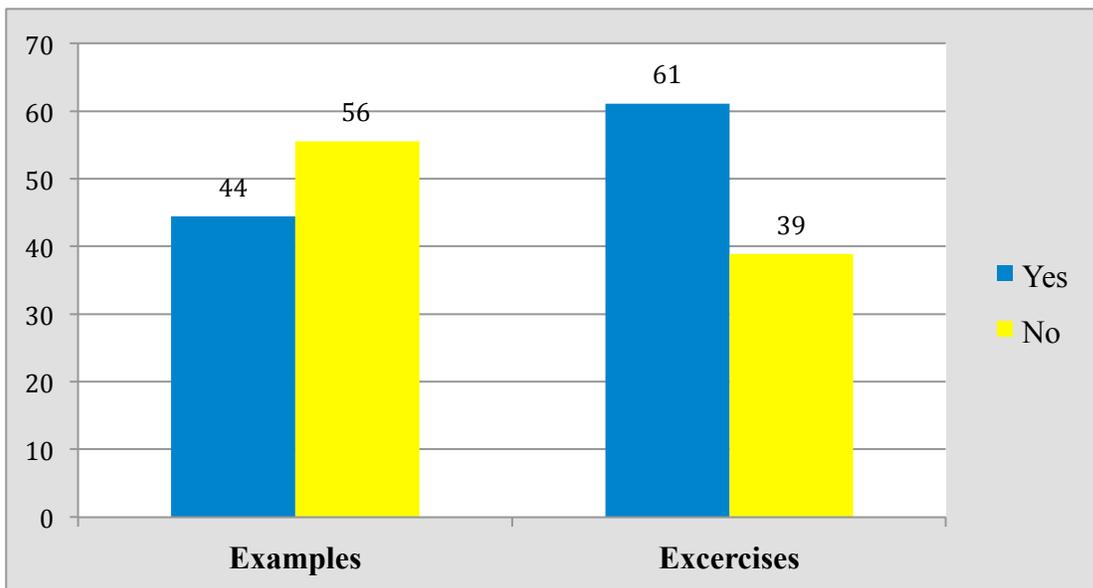
Many textbooks had multiple representations of functions. As depicted in Figure 1, the results highlight that tables were the most popular means to represent functions in textbooks (72%); while only 25% used an equation. None of the mathematics textbooks used mapping to represent functions. Furthermore, only 3% of the textbooks had no representation of functions. Figure 1 depicts the percentage of textbooks that incorporated the various representations.

Figure 1 Percentage of functions representations visible in textbooks.



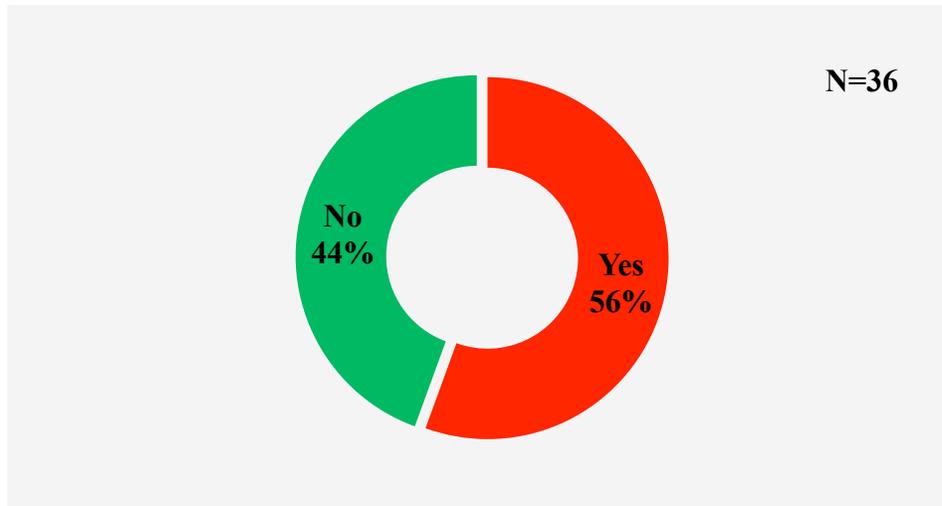
Furthermore, secondary mathematics textbooks during the period 1908-1950 represented the concept of function using real world context more readily in the exercises than in the examples. Figure 2 illustrates that 61% of the textbooks had a real world context in the exercises, while only 44% had a real world context in the examples.

Figure 2 Percentage of functions presented in real world situations in the example and exercises.



Additionally, some books unequivocally stated how to read a functional notation. For example, Crenshaw, Simpson, and Pirenian (1932) wrote, “The symbol does not mean f times x ; it is an abbreviation of the phrase “a function of the variable x ” and is read “ f of x .”” (p.123). How to interpret functional notation were presented in slightly more than half (56%) of the selected sample. Figure 3 illustrates the percentage of textbooks that describes how to read functional notations.

Figure 3 Percentage of textbooks that provided explicit guidance to read functions.



Importance of Functions in Secondary Mathematics Textbooks

During the period 1908-1950, textbooks were not likely to present functions as an independent topic in the Table of Contents. Table 3 reveals that only 17% of our sample had functions as an independent topic (Functions/ Functionality) in the Table of Contents.

Moreover, functions were likely to appear in the first half of the textbooks (Figure 4), and five pages or less were generally allocated explicitly to discuss the functional concept (Figure 5).

Table 3. Chapters in Table of Contents in which functions appeared

Titles of Chapters in Table of Contents	Number of Books (N=36)
Functions and graphs, functions and variables, functions and something	11
Functions/functional relations	6
Quadratic equations	4
Equations	2
Graphs	2
Graphic solution	2
Graph of linear equations	2
Graphical representation of function and equations	1
Linear function	1
Ratio, dependence, proportion	1
Algebraic expression	1
Variables and limits	1
Difference equations and generating functions	1
Simultaneous equations, graph	1

Figure 4 Percentage of appearance of functions in quartiles of secondary mathematics textbooks.

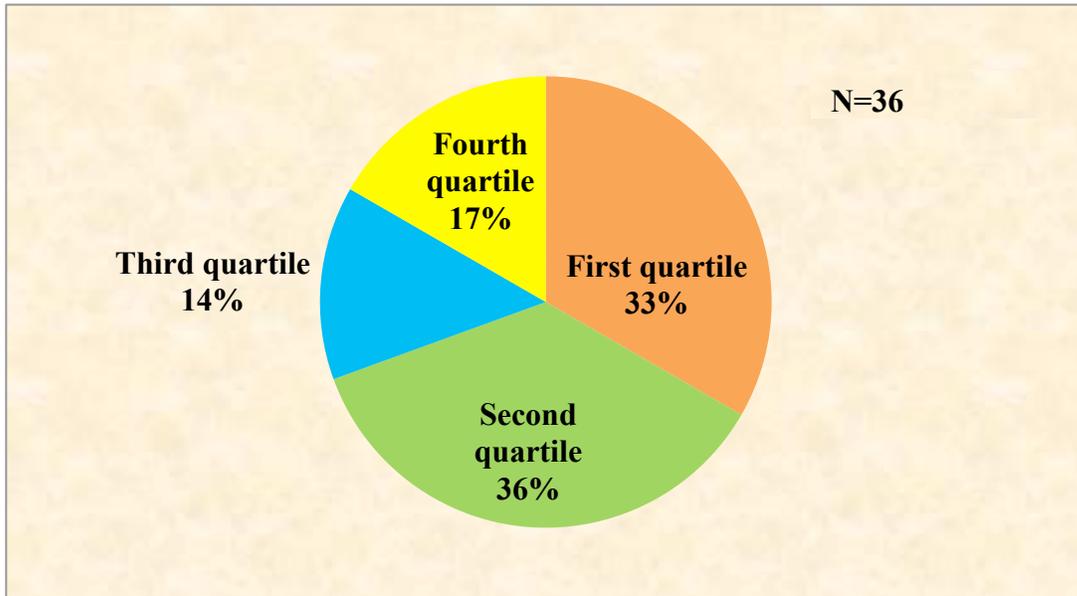
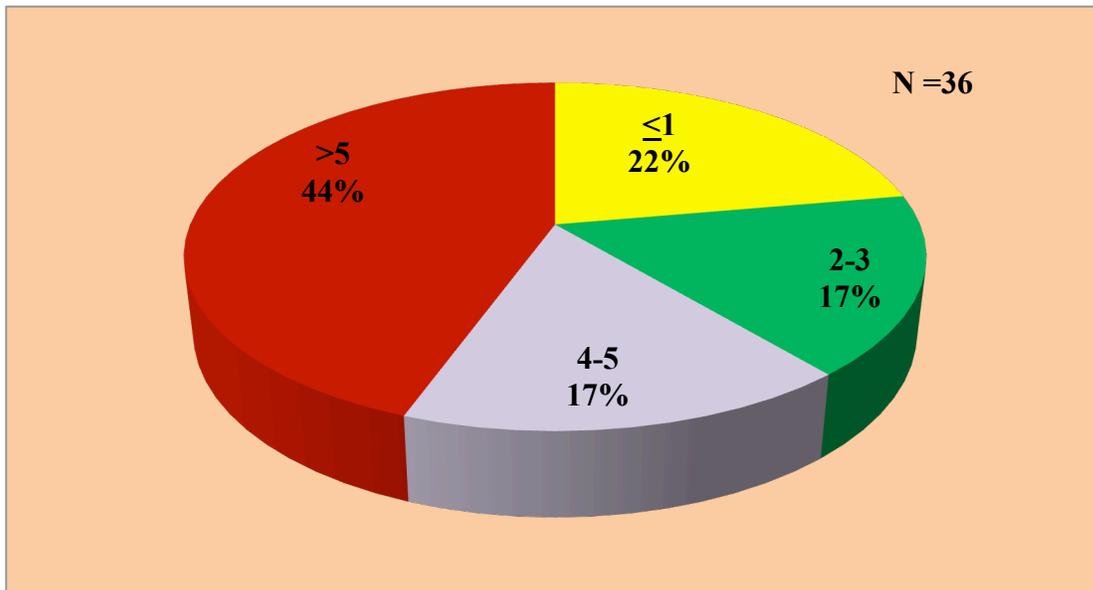


Figure 5 Percentage of number of pages allocated for the concept of function.



SUMMARY

Overall, there exist various definitions and representations of functions in school mathematics textbooks during the period 1908-1950. Furthermore, although functions

were placed in the first half of most textbooks, the concept of function was primarily allotted five or less pages.

The definitions of functions in historical textbooks embodied ancient and modern views. The various definitions aligned with Descartes notion of two dependent variable quantities, Leibnitz and Bernoulli analytical expressions, and Dirchlet correspondence relations between and independent and dependent variable (Kleiner, 1989; Ponte, 1992; Young, Denton, & Mitchell, 1911). Notwithstanding, two textbooks sought to differentiate cases for operational definitions of multivalued functions; the premises reinforced the relations between independent and dependent terms.

The usage of multiple representations to describe the function concept helps students learn mathematical ideas (Brenner, Mayer, Moseley, Brar, Duran, Reed, and Web (1997). Brennen et al. (1997), found that pre-algebra students who learned functions using multiple representations and had meaningful context performed better than students in a control group. Hence, the decision to utilize multiple representations of functions in the textbook during 1908-1950 might have positively influenced students learning of the concept of function.

Finally, the few times that functions appeared as an independent topic, and the average allocation of five or less pages in secondary school mathematics textbooks, reflect the cries that were made to increase functions visibility in school mathematics by The Reorganization of Mathematics in Secondary Education Report (1923) and The Report of Progressive Education and Joint Committee (1940). Although the number of pages allocated to the functions concept was miniscule at the turn of the century, it is apparent that progress was being made.

In conclusion, functions have become more prevalent in mathematics textbooks since the period 1908-1950. Taking into account the increased popularity of functions in school mathematics since 1950, future research needs to examine the concept of function in US secondary mathematics textbooks after the “New Math” era. Moreover, considering the importance placed on functions by the Common Core State Standards for Mathematics (CCSSM), studies ought to document the impact of the CCSSM on the concept of functions in secondary mathematics textbooks; as well as compare the emphasis placed on functions in US mathematics textbooks to other high-performing TIMSS or other international assessment countries. Such variation in historical mathematics textbooks, invokes a need to explore how function is presented in textbooks.

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