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The Problem-Solving Process in a Mathematics Classroom

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Problem solving provides a working framework to apply mathematics, and well chosen mathematics problems provide students with the opportunity to solidify and extend what they know, and can stimulate students' mathematics learning (NCTM, 2001). Using this framework, students may utilize ways to learn mathematics concepts and skills that are rich with meaning and connections, and pre- and in-service teachers may implement teaching and assessment procedures to establish teaching and assessment environments.

The Four-step Process

Polya's (1957) four-step process has provided a model for the teaching and assessing problem solving in mathematics classrooms: understanding the problem, devising a plan, carrying out the plan, and looking back. Other educators have adapted these steps, but the essence of these adaptations is very similar to what Polya initially developed. The following are two possible variations of Polya's model I have come across: (1) define the problem, develop a plan, implement the plan, and evaluate; or (2) plan, do, act, and check. The implementation of these steps in the classroom is not easy, and, in some cases, could be misused or misleading. In this article, after a discussion of the four-step model and possible connections to the *Common Core State Standards – Standards for Mathematical Practice (CCSS-SMP)* (NGACBP & CCSSO, 2010), I present three possible challenges in implementing this model in K-12 classroom as well as pre- and in-service teachers.

Understanding the problem. At this point, the student should try to understand the problem. This is the step where you want students to engage with the problem or task and want

to actually solve it. These are some possible questions the student could ask (Polya, 1957): Do I know what is the unknown? Do I know what are the data? Do I know what is the condition or conditions involved in the problem? In order to determine the unknown, is the condition sufficient, not sufficient, or redundant? Can I draw a picture to help you understand the problem? Can I introduce suitable notation? Can I separate and write down the various parts of the condition?

On the other hand, pre-service or in-service teacher may ask similar questions to facilitate or assess the understanding of the problem solving process: How has the student demonstrated that she knows what the unknown is? Is the demonstration correct and sufficient? Does she know what are the data and has used the data properly to understand the problem? Can she provide a description of or paraphrase the condition or conditions involved in the problem? In order to determine the unknown, can she identify the condition or conditions as sufficient, not sufficient, or redundant? Can she analyze givens, constraints, relationships, and goals (NGACBP & CCSSO, 2010)? Has the student drawn a picture to help her understand the problem? Has the student considered any suitable notation needed to understand the problem? Has the student separated and written down the various parts of the condition? Are there any specific hints I can provide to help the student understand the problem without giving away the possible plan or answer? Are there any other probing questions I could ask her to develop understanding of the problem and move to the next stage of the process? Are there any resources, materials, information I need to make accessible to the student? Are there any misconceptions or weaknesses related to the content of the problem (including social and experiential background)? Can she start by explaining to herself the meaning of a problem and looking for entry points to its solution and how can I facilitate this process (NGACBP & CCSSO, 2010)? Has she

considered analogous problems, and try special cases and simpler forms of the problem in order to gain insight into its solution (NGACBP & CCSSO, 2010)? NGACBP and CCSSO also indicated that,

Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches (par. 2).

Devising a plan. At this point, the students should try to find connections between the data and the unknown, consider auxiliary problems in an immediate connection cannot be found, and should eventually obtain a plan of the solution (Polya, 1957). Possible questions that the student could ask (Polya): Have I seen the same, similar, or related problem before? Do I know a theorem that I can use? Can I use the results, methods, or strategies of a similar problem? Could I restate the problem? Do I know the vocabulary involved in the problem? Could I solve a more accessible, general, analogous, or special problem? Could I simplify the problem? Could I derive something useful from the data, or think of other data appropriate to determine the unknown? Could I change the unknown or data, or both if necessary? Did I use and taken into account all

the data, essential notions or whole condition? Is the student using improper shortcuts, information or steps to solve the problem?

As before, a pre-service or in-service teacher may ask similar questions to facilitate or assess the student's efforts in devising a plan to solve the problem: How can I help her to see connections with a similar, or related problem? Is there a need to review definition of terms or theorems? Should I remind students of useful results, methods, or strategies of a similar and simpler problem? Can she restate the problem in her own words? Should she solve a more accessible, general, analogous, or special problem? Should I provide some guidance to help her derive something useful from the data, or think of other data appropriate to determine the unknown? How can help the student use and take into account all the data, essential notions or whole condition? Should the student go back to the previous stage of problem solving and try to have a better understanding of the problem? Can she make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt (NGACBP & CCSSO, 2010)?

Carrying out the plan. The student then carries out the plan developed in the previous step. The students should check each step of the solution plan. Possible questions are the following (Polya, 1957): Can I see clearly that the step is correct? Can I prove that it is correct?

Similarly, the pre- and in-service teachers may ask the following questions: What questions could I ask about carrying out the plan and make sure she sees clearly that the step or proof is correct? Should the student go back to the previous steps of the process and check understanding of the problem or the feasibility of the plan? As indicated in the *CCSS-SMP* (NGACBP & CCSSO, 2010), can she make sense of the problem and persevere in solving it?

Looking back. The students should examine the solution obtained. Possible questions are the following (Polya, 1957): Can I check the result, or argument? Can I derive the result differently? Can I see it at a glance? Can I use the result, strategy or method for another problem? Can she understand the approaches of others to solving problems and identify correspondences between different approaches (NGACBP & CCSSO, 2010)?

The pre- and in-service teachers may ask the following questions: Has she checked the result, or argument in a convincing and appropriate manner? Can she derive the result or present another argument? How can she use the result, strategy or method for another problem? How can I make the problem more realistic or general? Can she see the value of solving the problem in different manners?

Furthermore, Polya's model is very useful in the problem solving process as students solve mathematics problems, but it is also very useful in the teaching process. For example, as a teacher assess a students' problem solving solution process, she might notice that the students is working at devising a plan stage of the process. In this case, the teacher could facilitate the process by providing help at that stage, but should not get involved with the carrying out the plan stage, which is next.

Possible challenges in implementing this model

First, we need to keep in mind that learning this four problem-solving steps might not be sufficient to become a better problem solver or mathematician. This approach is mainly a working framework for problem solving. In my experience, an effective problem solver has the ability to work on a problem with flexibility, and following a linear four-step approach might not work all the time. For example, in some cases, you might start to solve a problem without a complete understanding of the problem and this should not stop you from trying to find a

solution. In my opinion, only textbook word problems are found already neatly set up for you. Most real life problems are presented as situations in very messy and random manners. In this type of setting, you might try to solve the “problem” and as you try to solve the “problem” you start understanding the problem. The “problem” evolves and changes as you try to solve it. Slowly, you start realizing what the problem is as you try to understand and solve it. For example, you want to go to college, how do you make it happen? You want to buy or rent an apartment, how do you solve this problem?

Second, sometimes the four-step problem solving process is more useful when you start to organize your arguments. Intuitively, you might feel you know the solution to a problem, but you still need to convince others that you have the correct method or answer. Polya’s (1957) steps could be used to make sure you can present an acceptable and convincing argument. For example, a lawyer might believe the innocence of the client intuitively, but he/she still needs to prove the innocence of the client to the judge or jury. This will involve a proper understanding of the case and laws involved in the case, the development of a proper defense plan, carrying out the defense by presenting arguments and evidence in favor of the client, and looking back to see if more evidence is needed, which in some cases might be arguing for a mistrial, or new trial. Another, example is when a medical doctor believes he/she knows the nature of an illness afflict a patient, but he/she still needs to run some labs to make sure the diagnosis is correct.

Third, the four-step approach could become too methodical or too linear, and might prevent students from being more creative, and to think “out of the box.” As stated in *CCSS-SMP* (NGACBP & CCSSO, 2010), it important that students “monitor and evaluate their progress and change course if necessary” (par. 2). This lack of flexibility could be avoided by

visualizing the problem solving steps as parts of a puzzle (see figure 1). In this view, each part is complementing each other like in a puzzle.

Figure 2 presents another possible diagram of the four areas of the problem-solving process (Polya, 1957) with suggested strategies for each phase. Students could use it to facilitate their problem solving process. Teachers could use it to check the presence of the strategies as students work on problems, provide appropriate hints and guidance, and assess their understanding and progress. The areas are interrelated and should not be used in a linear fashion. Instead, as the arrows illustrate, you could move back and forth from one area to another as needed to solve a problem.

Fourth, teaching of specific problem-solving strategies in isolation could hinder the development of flexibility and the problem-solving process could become an exercise instead. I try to avoid this approach. Instead of teaching specific strategies, I try to help students work with the problem-solving process and the connection between problems. As indicated by Schoenfeld (1985), it is possible to teach learners to use general strategies such as those suggested by Polya, but that is insufficient. It might take several revolutions through the four-step process before finding a solution to a problem.

Concluding Remarks

A true problem solving process will allow students to be flexible, intuitive, and creative. The students should be allowed to move from one step to another, and through many alternatives and strategies. The teachers will also need to be flexible in their assessment of the students and provide many opportunities for discovery and exploration. Finding great problem-solving situation is a challenge, but it is crucial in we want to be effective.

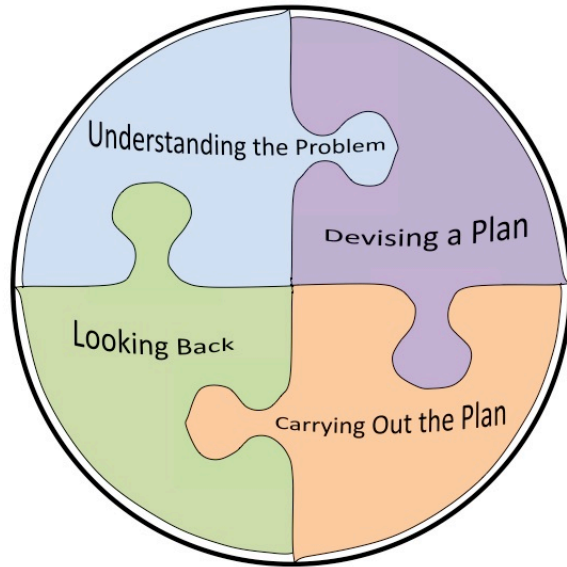


Figure 1. Problem-solving process viewed as a puzzle

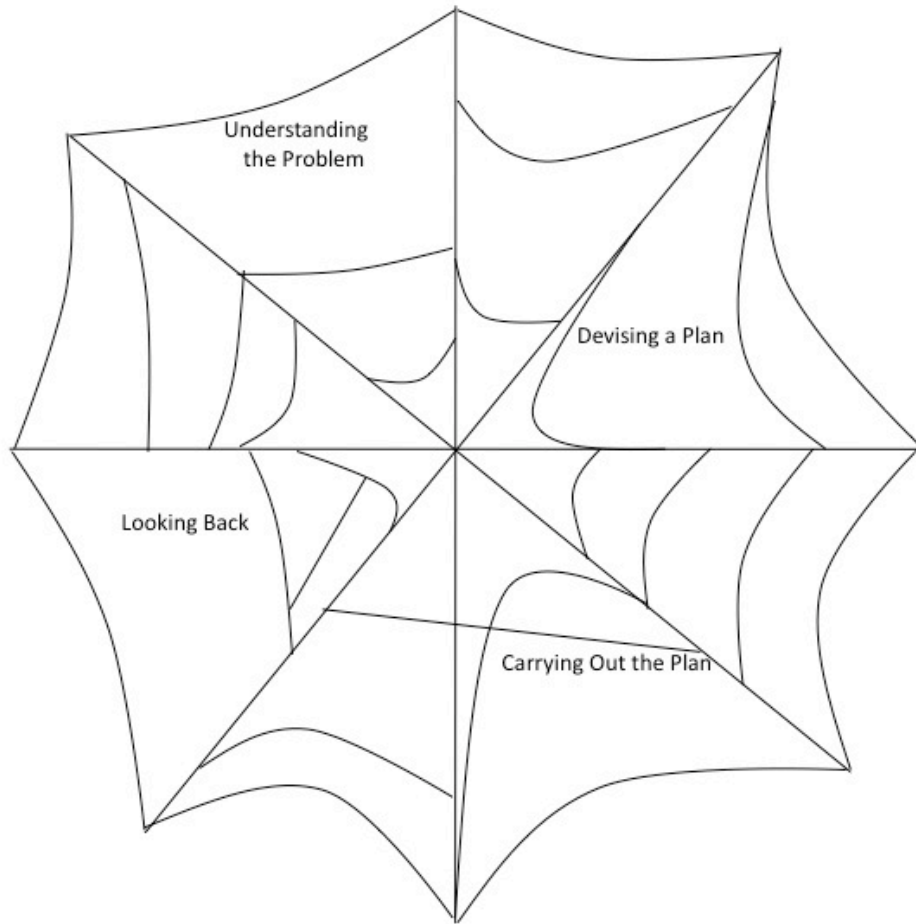


Figure 2. Problem-solving process viewed as a web

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