Physics Lab Experiments with Simulated Data for Remote Delivery

Department of Chemistry and Physics

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Experiment 1.01: Measurement

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I. EXPERIMENT 1.01: MEASUREMENT

A. Abstract

The linear dimensions of a rectangular prism (cuboid) and right circular cylinder are measured, as well as their masses.

B. Formulas

$$V_{\text{rect.prism}} = lwh \tag{1}$$

$$V_{\text{cylinder}} = \frac{1}{4}\pi D^2 l \tag{2}$$

$$\rho = \frac{m}{V}, \ \text{ average density} \tag{3}$$

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N}, \ \text{ mean} \tag{4}$$

$$\delta x = \sigma_{N-1} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}}, \ \text{ sample standard deviation of the mean} \tag{5}$$

$$\delta\bar{x} = \frac{\delta x}{\sqrt{N}}, \ \text{ standard deviation of the mean or standard error} \tag{6}$$

$$\text{\%-Err}\,(x_{\text{exp}}) = \frac{|x_{\text{exp}} - x_{\text{accepted}}|}{x_{\text{accepted}}} \times 100 \tag{7}$$

$$\text{\%-Diff}\,(x_{\text{exp}}) = \frac{|x_{\text{exp1}} - x_{\text{exp2}}|}{\left(\frac{x_{\text{exp1}}+x_{\text{exp2}}}{2}\right)} \times 100 \tag{8}$$

C. Description and Background

In this lab you will be introduced to simple experimental data analysis methods and the use of significant figures. You will need to find the volumetric density of a right circular cylinder and its experimental error. You will also need to compare your result with the accepted value for the density of the material used. Additionally, you will also learn to use a Vernier caliper for length measurements.

1. Measurement Uncertainty

There is uncertainty, or error, associated with a measurement (*i.e.*, the process of recording the observations associated with physical, and especially continuous, parameters). Some eror, like personal and systematic (equipment bias), is avoidable. Some error is unavoidable. Random error is the result of the practical inability to prepare and execute an experiment exactly the same way every time. Assuming the variations are unbiased and normally distributed, one can compensate by repeating the measurement.

Continuous measurements are nominally represented by a finite string of numbers, arrived at with some degree of certainty (the so-called significant digits or figures), followed by an endless string of uncertain numbers. The uncertain places in the decimal expansion are left *unfilled.* The last (right most) digit is the most uncertain. Pure numbers (non measurements) are infinitely certain.

Analog readout measurement devices, such as rulers, involve counting increments (scale subdivisions) down to the smallest ones and possibly estimating a fraction thereof. The uncertainty is one of these smallest increments or a fraction of it. For example, in the figure above, the measurement might be

$$\ell = 4.4 \ units$$

This measurement has two significant figures since it is shorthand for

$$\ell = 4.4??????? \cdots \ units$$

The "4" in the tenth place is the most uncertain significant digit. The uncertainty can be estimated as a reasonable fraction of the smallest increment, so, for example,

$$\delta\ell = 0.1 \ units$$

Depending on the scale resolution, or size of the scale's smallest subdivision, a reasonable uncertainty can be bigger or smaller. The complete measurement would then be reported

$$4.4 \pm 0.1 \longleftrightarrow [4.3, \ 4.5]$$

An uncertainty should not be reported with any more than one significant figure.

Digital (mechanical or electronic) readout measurement devices remove the factional estimation associated with analog devices. For example, the electronic digital readout of a centi-gram mass scale might be

$$m = 7.54 \ g$$

With such instruments the uncertainty is *one* smallest output unit, therefore in the example

$$\delta m = 0.01 \ g$$

2. Significant Figures

Derived quantities based on direct uncertain measurements must be consequently uncertain as well but in a consistent way. When two uncertain measurements are multiplied or divided, the resulting derived quantity is no less uncertain than the more uncertain input/ingredient. For example, if the distance a ball drops is $d = 1.55 \ m$, and the travel time is $t = 0.56 \ sec$. The average speed is

$$v = \frac{d}{t} = 2.8 \ m/s$$

Note that the derived quantity, the speed, has no more than two significant figures as the time does. When uncertain measurements are added or subtracted, the result cannot be certain beyond the first (left to right) least-certain place. So all measurements are rounded off to this place before the operation, For example,

$$
\begin{array}{ll}
42.31 & 42.3 \\
\quad 0.0621 & \quad 0.1 \\
512.4 & 512.4 \\
+ \ 2.57 & + \quad 2.6 \\
\hline
& 557.4
\end{array}
$$

Among these four measurements, the first least-certain place is the tenth's place.

It is possible, for example, in statistical determinations of the uncertainty or standard error, that the least certain place in the measurement or derived quantity does not coincide with that of the uncertainty. In such a case, the two must be brought into line since they are added/subtracted. For example, if the average value of the acceleration due to gravity in a repeated experiment is

$$\bar{g} = 9.75 \ m/s^2$$

but the associated standard error works out to be

$$\delta\bar{g} = 0.1 \ m/s^2$$

then the results should be reported as follows

$$(9.8 \pm 0.1) \ m/s^2$$

3. Error Propagation

There is a mathematical way to determine the uncertainty in a derived quantity based on the uncertainties of the directly measured parameters. This process is referred to as propogation of error. In cases in which the derived quantity is the product of powers of N parameters, $i.e.$,

$$Q = p_1^{n_1} p_2^{n_2} \cdots p_N^{n_N}$$

then

$$\delta Q = Q\sqrt{\left(n_1\frac{\delta p_1}{p_1}\right)^2 + \left(n_2\frac{\delta p_2}{p_2}\right)^2 + \cdots + \left(n_N\frac{\delta p_N}{p_N}\right)^2}$$

where δa and δb are the uncertainties in the directly measured parameters, a and b, respectively, and δQ is the consequent uncertainty in the derived quantity.

4. Statistics

The experiments in the physics laboratory will involve measurements that can be repeated endlessly in principle but not in practice. Therefore any sequence of measurements represents a sample for which one can compute a sample mean and a sample standard deviation of the mean. The standard error is the standard deviation of a distribution of samples and is

approximated as

$$\delta\bar{x} \approx \frac{\delta x}{\sqrt{N}}$$

where $\delta x \equiv \sigma_x$, the sample standard deviation of the mean.
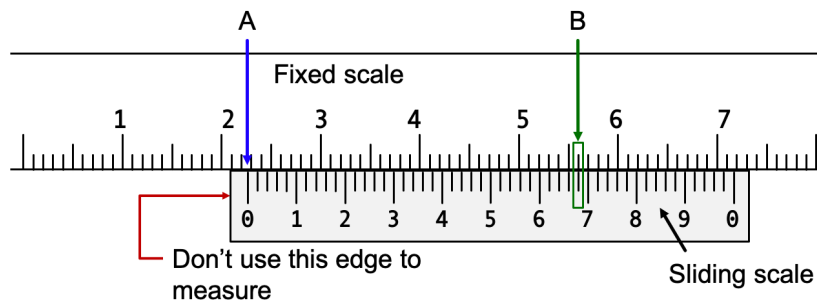
5. *Vernier Calipers*
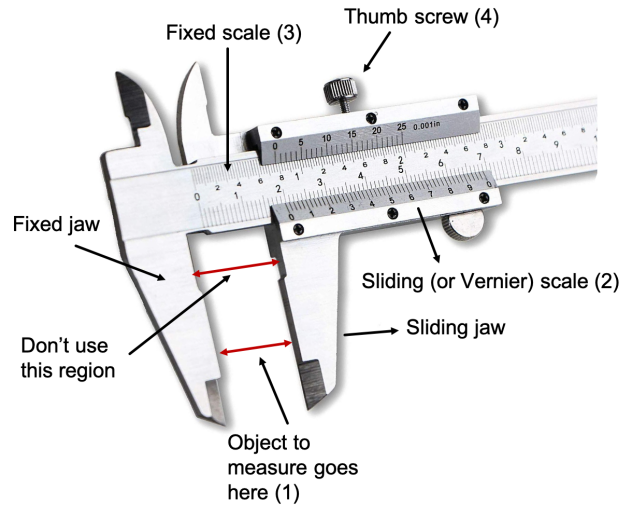
A Vernier caliper is used to measure dimensions to within thousandths of a centimeter. We will use the Vernier caliper in a couple of experiments, so it is important that you learn to read the scale properly. Remember that a small error in measurement may greatly affect subsequent calculations. The figure shows a typical Vernier caliper. You will use the larger "jaws" (fixed and sliding) most frequently to measure outer dimensions, such as the width of a block or circular cylinder.

Most Vernier calipers have both metric and English scales; make sure that you read the correct scale. The fixed portion of the scale (3) is marked in increments of 0.1 cm; the sliding scale (2) has numbers marking 0.01 cm increments, and small lines marked in 0.002 cm increments.

6. *How To Measure*

1. Loosen the thumb screw clamp (4), if necessary, and close the sliding jaw so that it fits on the object to be measured. If the object is circular or spherical, make sure you're

A   B

Fixed scale

1   2   3   4   5   6   7

Don't use this edge to
measure

0   1   2   3   4   5   6   7   8   9   0

Sliding scale

measuring at the widest point. Make sure to use the flat section of the caliper jaws (1), not the region marked in the figure.

2. Gently tighten the thumb screw clamp and remove the caliper from the object; the screw will allow you to move the caliper without changing the position of the sliding jaw.

3. Now look at the scale on the sliding jaw, as shown in the magnified image of this region (see figure).

4. Take the first portion of the measurement by looking at the line on the fixed scale, above the left zero on the sliding scale (A). In this example, this line falls between the 2.2 and 2.3 cm lines. Our measurement falls within this range, so we'll start our measurement with 2.2 cm.

5. Make sure your measurements are taken from point (A), the line directly below the first zero on the sliding scale, and not from the edge of the sliding scale, otherwise your reading will be off.

6. The next two digits are found by carefully finding the line, on the sliding scale, that is straight across from a line on the fixed scale. Several may look as though they line up, but only one will match best.

7. In the example above, we see that the second small line to the right of 4, on the sliding scale is straight across from a line on the fixed scale, at point (B). The last two digits of your measurement come from the sliding scale, not the fixed scale. Since the lines on the sliding scale are 0.002 cm apart, the line at point (B) represents 0.068 cm.

8. Now adding the total measurements together, you get 2.2 cm+0.068 cm=2.268 cm as the final reading.
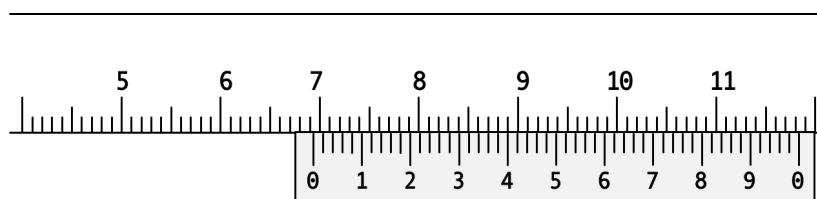
9. Notice that the last digit of your measurement will always be an even number, with the Vernier calipers used in this course.

7. Practice

Confirm the readings in these examples:
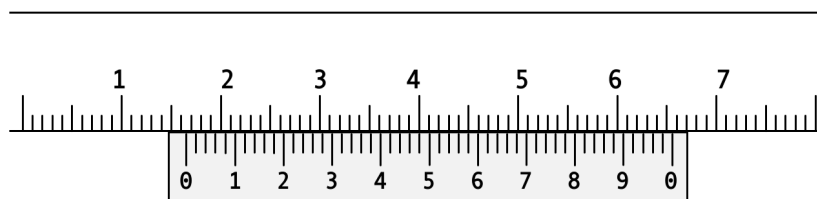
(a) 1.648

(b) 6.940

D.   Procedure

1.   Part I.

1. Measure the length, width, and height of the aluminum rectangular prism using Vernier calipers. Make the trial measurements along different sections of each side.

2. Determine the mass of the rectangular prism.

2.   Part II

1. Measure the length and diameter of the right circular cylinder.

2. Determine the mass of the cylinder.

E. Measurements

		cuboid		
trial	length: l [cm]	width: w [cm]	height: h [cm]	mass: m_{cub} [$gram$]
1				
2				
3				

		cylinder	
trial	length: l [cm]	diameter: D [cm]	mass: m_{cyl} [$gram$]
1			
2			
3			

F. Instructions

1. Make sure the standard error is rounded to one significant figure and that its precision is consistent with that of the average density's.

2. Calculate the rectangular prism's volume and density (to the appropriate number of significant figures), and compare the calculated density with the accepted value for aluminum.

3. Calculate the cylinder's volume and density (to the appropriate number of significant figures), and compare the calculated density with the accepted value for aluminum.

4. Using the data for the cylinder's length, l, calculate the average length (in cm) following the rules for significant figures.

5. Using the data for the cylinder's diameter, D, calculate the average radius (in cm) following the rules for significant figures.

6. Using the data for the cylinder's mass, m_{cyl}, calculate the average mass (in $grams$) following the rules for significant figures.

7. In the cylinder case, use the averages calculated above for the length and diameter, $\bar{l}$ and $\bar{D}$, as well as the average value for the mass, $\bar{m}_{\text{cyl}}$, to determine $\bar{\rho}_{\text{cyl}}$ (this is a different procedure than was followed in the table above).

8. The uncertainty in the density can be determined in the following, alternative way. Take the instrumental limit of error associated with the Vernier calipers used to be the least count or smallest measurable increment

$$\delta l_{\text{Vernier}} = 0.002 \; cm$$

Assume the uncertainty in all linear measurements to be the same and equal to this value, $i.e.$, $\delta l = \delta D = \delta l_{\text{Vernier}}$. The propagated error formulas for volume and density can be combined so that

$$\left(\frac{\delta \rho_{\text{prop}}}{\bar{\rho}} \right)^2 = \left(\frac{\delta m}{\bar{m}} \right)^2 + \left(\frac{1}{\bar{l}^2} + \frac{4}{\bar{D}^2} \right) \delta l_{\text{Vernier}}^2 \tag{9}$$

If the uncertainty in the mass measurement ($i.e.$, the mass balance's limit of error) is taken to be $\delta m = 0.1 \; g$, use Eq. (9) to determine $\delta \rho_{\text{prop}}$

G. Calculations

	cuboid	
trial	volume: V [m^3]	density: ρ [kg/m^3]
1		
2		
3		

average density, ρ_{cub} [kg/m^3]	
standard deviation, $\delta\rho_{\mathrm{cub}}$ [kg/m^3]	
standard error, $\delta\bar{\rho}_{\mathrm{cub}}$ [kg/m^3]	
accepted density of Aluminum, ρ_{accepted} [kg/m^3]	2700
%-Err (ρ_{cub})	

	cylinder	
trial	volume: V [m^3]	density: ρ [kg/m^3]
1		
2		
3		

average density, ρ_{cyl} [kg/m^3]	
standard deviation, $\delta\rho_{\mathrm{cyl}}$ [kg/m^3]	
standard error, $\delta\bar{\rho}_{\mathrm{cyl}}$ [kg/m^3]	
accepted density of Aluminum, ρ_{accepted} [kg/m^3]	2700
%-Err (ρ_{cyl})	

$\bar{l}_{\mathrm{cyl}}$ [cm]	
$\bar{D}_{\mathrm{cyl}}$ [cm]	
$\bar{m}_{\mathrm{cyl}}$ [g]	
$\bar{\rho}_{\mathrm{cyl}}$ [kg/m^3]	
$\delta\rho_{\mathrm{prop}}$ [kg/m^3]	